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Option Pricing Analysis with Implied Volatility

Analýza oceňování opcí s implikovanou volatilitou

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4. Comparative Analysis Using Real Market Data
5. Conclusion

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The declaration

"Herewith I declare that I elaborated the entire thesis, including all annexes, independently. "

Ostrava dated......20. 4. 2016...........

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Content

1. Introduction

It is undeniable that derivatives are becoming increasingly popular and important in financial market. Nowadays, options are traded actively on the exchange-traded markets and in the over-the-counter market all over the world. Options give the holder the right to do something, which is different from other derivatives and makes options attractive to the traders. One of the most important information that the traders concern is the price of financial instrument. How can we price options? What factors can influence the prices of options?

The goal of the thesis is to examine the existence of implied volatility smile and construct implied volatility surface by using the real market data, and focus on pricing stock option and lookback option by using implied volatility which can be found in implied volatility surface.

In Chapter 2, we will introduce the basic characteristics of options, which includes the terminology of options and the types of options. Then we will present some knowledge about stock option pricing, where put-call parity, risk-neutral pricing theory and non-arbitrage pricing principle will be laid emphasis on in this part. One of the most important part in Chapter 2 is the introduction of Black-Scholes-Merton model, which is the fundamental pricing model for stock options. The stochastic process for stock price, Itô's lemma and the Black-Scholes-Merton differential equation are introduced for a better understanding of the Black-Scholes-Merton model. After obtaining all these information, we can then derive one of the most significant equation in the thesis, the Black-Scholes pricing formula. In the end of this part, we will extend the Black-Scholes pricing formula to pricing options on paying dividends. And we will lay emphasis on the pricing of lookback options in the rest part of Chapter 2, the pricing formula for floating lookback option and fixed lookback option will be introduced respectively.

We will introduce the concept of implied volatility in Chapter 3. Firstly, we will compare implied volatility with the historical volatility and introduce how to calculate the implied volatility. Then we will present CBOE volatility index (VIX), which can reflect the expectation of the volatility in the market. The most significant essential in the thesis is the idea of implied volatility smile, which will be highlighted in Chapter 3. In this part, we will not only explain why volatility smile is the same for both call and put options, but the cause for the existence of volatility smile, especially for foreign currency options and equity options, will be presented as well. Last but not the least, we will introduce the term structure of implied volatility and the construction of implied volatility surface, which can connect implied volatility smile and its term structure together.

Chapter 4 is the application part of the thesis. In Chapter 4, we will calculate the value of implied volatility of AAPL stock options with various exercise prices and various expiration dates by applying the real market data. After obtaining implied volatilities, we will examine the existence of implied volatility smile and its term structure and then construct the implied volatility surface. In order to accomplish the goal of the thesis, we will use the implied volatility, which can be found in implied volatility surface, to price both stock option and lookback option, and the results will be compared with prices calculated using both the historical volatility and ATM volatility. Furthermore, we will also construct the option price surface to illustrate the relationship between option price, implied volatility and time to maturity.

In the last chapter, we will summarize all the results we discussed before and make a clear conclusion for the thesis.

2. Options and their Pricing

Financial derivatives have developed an increasingly significant role in financial market throughout the world. Options, as one of the most important instruments of financial derivatives, are traded actively on both exchange-traded markets and in the over-the-counter market all over the world nowadays.

2.1. Terminology of Options

In this chapter, most of the descriptions are based on the information from Hull (2009, p179).

Options are kind of financial instruments whose value derives from its underlying assets. In general, options give the holder the right but not the obligation to buy or sell the underlying assets at specific dates for specific prices. In terms of the rights, there are two types of options, which respectively are call options and put options. As Hull (2009, p179) illustrates that "*A call option gives the holder the right to buy the underlying asset by a certain date for a certain price. A put option gives the holder the right to sell the underlying asset by a certain date for a certain price.*"

In the options contracts, the specific date in the contract is known as maturity date or expiration date; the certain price in the contract is the strike price or the exercise price. Options can be classified into two groups by considering whether it can be exercised on any date before the expiration date or not. According to this classification, the American options can be exercised at any time before or on the expiration date, while the European options can be only exercised when it matured. Moreover, most of the options that are traded on the exchange-traded market nowadays are American options.

For each option contract, there are two sides of investors: for the investors who have bought options, it is said that they take the long position; for the investors who have written or sold options, it is said that they take the short position. As a result, there are four types of options positions, which respectively are a long position in a call option; a short position in a call option; a long position in a put option and a short position in a put option.

2.2. Types of Options

According to the difference in the underlying assets of options, it can be classified into different types. And the most of descriptions in this chapter are based on the information stated by Hull (2009) and Haug (2007).

Stock options The value of this kind of options derive from the prices of its underlying stocks and these options are primarily traded on exchange-traded markets. The exchange-traded market is kind of financial market that allows individuals to trade standardized contracts which have been defined by the exchange. The main exchange-traded markets in America are the Chicago Board Options Exchange, the American Stock Exchange, the International Securities Exchange, the Boston Options Exchange and the Philadelphia Stock Exchange. The stocks are normally traded in the size of 100 shares, hence, for the stock options, an option contract gives the holder the right to transact the unit of 100 shares in general.

Exotic option This kind of option refers to a collection of the option products with nonstandard contracts that are traded on the over-the-counter market. The over-the-counter market is a significant alternative to exchange-traded market. The traders of options complete its trading process over the telephones or through computer-linked networks on the over-the-counter market. The over-the-counter market allows the traders tailor the exercise prices, expiration dates and other features in the contracts in order to meet certain needs of corporate treasures. Moreover, compared with exchange-traded market, the over-the-counter market also has the advantage of larger trading volume.

Exotic options are more profitable than plain vanilla products, which have standard defined properties and trade actively, such as normal European and American call and put options. The exotic options are designed to meet specific needs of dealers, for example, in order to hedge the risk in the market, and reflect potential future movements in market variables. One of the most appealing exotic options for investors is the lookback option.

Lookback option The payoffs of lookback options can be earned depend on the maximum or the minimum underlying assets price achieved during the life of the option. The lookback options can be classified into two categories, which are floating lookback option and fixed lookback option.

The floating lookback options have no exercise price. The holder of floating lookback call has the right to buy the underlying asset at the minimum price achieved during the life of the option. The payoff for a floating lookback call is the difference between the final asset price and the minimum asset price observed during the life of the option, which can be expressed by Equation (2.1).

$$c(S_T, S_{min}, T) = max(S_T - S_{min}; 0) = S_T - S_{min} \tag{2.1}$$

where S_T refers to the final asset price on the expiration date, S_{min} represents the minimum price observed during the life of the option, T; and c denotes the value of European call options.

Similarly, the holder of floating lookback put has the right to sell the underlying asset at the maximum price achieved during the life of the option. The payoff for a floating lookback put is the difference between the maximum asset price observed during the life of the option and the final asset price on the expiration date, this relationship can be described in the form of formula, which can be expressed in Equation (2.2).

$$p(S_T, S_{max}, T) = max(S_{max} - S_T; 0) = S_{max} - S_T \tag{2.2}$$

where S_{max} denotes the maximum asset price achieved during the life of the option and p represents the value of European put options.

In contrast to the floating lookback option, the exercise price is specified for the fixed lookback option. The payoff for a fixed lookback call is the maximum difference between the highest asset price achieved, S_{max}, during the life of the option and its exercise price, K, and 0, which can be described in Equation (2.3).

$$c(K, S_{max}, T) = max(S_{max} - K; 0) \tag{2.3}$$

Similarly, the payoff for a fixed lookback put is the maximum difference between the lowest assets price achieved, S_{min}, during the life of the option and the strike price, K, and 0, which can be described in Equation (2.4).

$$p(K, S_{min}, T) = max(K - S_{min}; 0) \tag{2.4}$$

2.3. Stock Options Pricing

The pricing process for stock options is the most common and fundamental pricing process and can be applied to price other kind of options as well. In order to understand how to price stock option, the conception of put-call parity, risk-neutral pricing theory and non-arbitrage principle must be comprehended firstly.

Most of the descriptions in this chapter are based on the information from Hull (2009).

2.3.1. Factors Affecting Stock Option Prices

Since the value of stock options derive from the value of its underlying assets, the factors affecting the price of stock option are highly connected with the factors affecting the price of its underlying stock. The price of stock option can be affected by six important factors, and the relationship between the movement of the factors and the movement of the price of options can be clearly described in Table 2.1.

The current stock price, S_0 and the strike price, K The payoff of a call option is equal to the amount that stock price exceeds the strike price, hence, the price of a call option will increases while the stock price increases and the strike price decreases. In contrast, the payoff of a put option is equal to the amount that strike price exceeds the stock price, therefore, the price of a put option will increases when the strike price increases and the stock price decreases.

The time to expiration, *T* The prices will increase, or at least keep the same, as the time to expiration increases in terms of both call and put American options, because the holder of long-life options will have more opportunities to exercise than the holder of short-life options. For the European call and put option it holds the same in general, but there exist exceptions. For example, the dividend paying will decrease the stock price, if the a huge amount of the dividend payment happens on the expiration date of long-life options, then the value of long-life option is supposed to be lower than the short-life options.

The volatility of the stock price, σ As Hull (2009, p202) has stated "*The volatility σ of a stock is a measure of our uncertainty about the returns provided by the stock.*" If the volatility of the underlying stock increases, its reflection will be the increasing of the probabilities that the stock price will behave very well or very poorly, and the prices of both call and put options will increase as well.

The risk-free interest rate, *r* If the risk-free interest rate increases, the expected return of the stock is likely to increases as well. However, the increases of risk-free interest rate will result in the decreases of the present value of future cash flow received by the investors. And these two effects will lead to the increase in the value of call options and the decrease in the value of put options, under the assumption that all other variables stay constant.

The dividends expected during the life of the option The expected dividend payment will result in the decrease of the stock price. As a result, it will lead to the decrease of the value of call options as the amount of future dividends increases; and the amount of future dividends increases will have a positive influence on the value of put options.

Table 2.1 Factors Affecting Prices of Stock Options

Factors	Change	Call option price	Put option price
Current stock price, S_0	↑	↑	↓
Strike price, *K*	↑	↓	↑
Time to maturity, *T*	↑	↑	↑
Volatility, σ	↑	↑	↑
Risk-free interest rate, *r*	↑	↑	↓
Dividend yield, *q*	↑	↓	↑

Source: Hull (2009, p202) with own arrangement.

2.3.2. Put-Call Parity

The put-call parity describes the relationship between the value of call option and put option, which has the same strike price and maturity date. Considering two different portfolio with the same value, one portfolio contains one European call option and an amount of cash, whose value equals t*o* Ke^{-rT}; the other portfolio contains one European put option and a share. The value of both portfolios are the maximum value between S_T and K. Since both portfolios hold the European options, which cannot be exercised before the expiration date, two portfolios must have identical values today. The relationship between these portfolios can be expressed in Equation (2.5).

$$\mathrm{c} + Ke^{-rT} = p + S_0 \tag{2.5}$$

The implication contained by Equation (2.5) is illustrated by Hull (2009, p208): "*It shows that the value of an European call option with a certain strike price and exercise date can be deduced from the value of an European put option with the same strike price and exercise price, and vice versa.*" This relationship shown in Equation (2.5) only holds for European options with no dividends payment, for American options, since it can be exercised at any day prior to the expiration date, the relationship between calls and puts can be revised as

$$S_0 - K \leq C - P \leq S_0 - Ke^{-rT} \tag{2.6}$$

Where C and P respectively denotes the value of American call options and the value of American put options.

If consider the impact of dividends by assuming the present value of the dividends is D during the life of the option, and two portfolios with the same value can be constructed as one with an European call option and an amount of cash, whose value equals to $D + Ke^{-rT}$; the other portfolio contains an European put option and a share. Therefore, the put-call parity can be expressed in Equation (2.7).

$$c + D + Ke^{-rT} = p + S_0 \tag{2.7}$$

Or if we assume the stock paying dividend yield at rate q for long-life options, then the put-call parity can be revised as:

$$p + S_0e^{-qT} = c + Ke^{-rT} \tag{2.8}$$

Moreover, for the American options on a dividend-paying stock, the relationship between call and put options must be modified to

$$S_0 - D - K \le C - P \le S_0 - Ke^{-rT} \tag{2.9}$$

Or

$$S_0 e^{-qT} - K \le C - P \le S_0 - Ke^{-rT} \tag{2.10}$$

2.3.3. Risk-Neutral Pricing Theory

There exist an important argument for options pricing process, which is known as risk-neutral pricing theory. When pricing the stock options, we can assume the investors are risk-neutral, which means the investors do not need additional expected return while the investment risk increases, namely, the investors are indifferent to the risk. When the expected return of asset is discounted at the risk-free interest rate, the risky assets and riskless assets are indifferent to the investors.

The conclusion of risk-neutral pricing theory proves that under the circumstance of no arbitrage opportunities exist, if the value of derivatives still depend on the tradable securities, then the prices of derivatives are irrelevant with the risk attitude of the investors.

A world with all the investors that are indifferent to the risk is defined as a risk-neutral world. The real world is not a risk-neutral world, because as the investment risk increases, the expected return that investors charged will also increase. However, the resulting prices of stock options are proved to be correct under the assumption of risk-neutral world not only in the risk-neutral world, but also in the real world.

The risk-neutral pricing theory shows that any assumptions about the upward movement probability and downward movement probability are unnecessary in order to derive the pricing formula for options. Moreover, the key reason is that the price of option is not in absolute terms, because the value of options are calculated in terms of prices of the underlying securities. Therefore, the probabilities of upward or downward movements in the future are already reflected in the stock price, which means it is unnecessary for us to take these probabilities into account again when the underlying assets of options are stocks. Hence, when deriving the pricing formulas for stock options, we can apply the assumption of risk-neutral world, and one of the most significant remaining assumption is the absence of arbitrage opportunities.

2.3.4. Non-arbitrage Pricing Principle

Arbitrage, also named spread trading, is defined as a kind of market activity of buying or selling securities, commodities, derivatives or currencies in one financial market, and simultaneously selling or buying it in another financial market, which will let traders profit from a temporary difference in the price. Arbitrage is the result of market inefficiencies, which is the situation that the market prices of securities are not priced accurately all the time, it means not all of the publicly available information are reflected by the market prices. Under this condition, the current prices tend to deviate from their true discounted value of their future cash flows.

The absence of arbitrage opportunities is one of the most important assumptions in order to calculate the risk neutral prices and derive the pricing formula for options. The situation of non-arbitrage opportunities is defined as the market prices do not allow for profitable arbitrage, which means there are no opportunities to make risk-free profit. Moreover, if the market is arbitrage free, then the law of one price will be held, which means an assets must be sold for the same price in all locations. In other words, we can make the conclusion that any two portfolios, which have the same payout at some future time *t*, must hold the same price at time *t=0*.

Considering the circumstance of option trading, under the assumption of the absence of arbitrage opportunities, the law of one price is also held. Hence, the price of call or put options have to be equal to the price of any portfolio which has the same payoffs in the same conditions. Therefore, the theory of non-arbitrage pricing for options focus on the construction of the replicating portfolio, which has the same future cash flow and payoff as the option itself. According to the law of one price, the current value of the replicating portfolio should be equal to the price of option.

In order to meet the conditions of non-arbitrage opportunities, there should exist some bounds for the prices of options so that the market is arbitrage free. For European call options on non-dividend-paying stocks, the calls gives the holder the right to buy a share of underlying stock at the exercise price, thus the value of calls cannot be greater than the stock itself. Therefore, the stock price is the upper bound for European calls. And the price of a European call on non-dividend-paying stock cannot be lower than the value of $S_0 - Ke^{-rT}$. By examining whether the option prices fall in the range, which is shown in Equation (2.11), can we find the existence of arbitrage opportunities or not.

$$\max(S_o - Ke^{-rT}, 0) \leq c \leq S_0 \tag{2.11}$$

And for the European put options on non-dividend-paying stock, since the value of European put cannot be worth more than exercise price, *K*, at the maturity date, then the current price of European put cannot exceed the present value of exercise price, which is equal to

Ke^{-rT}. Therefore, the upper and lower bounds for European put options is:

$$\max(Ke^{-rT} - S_0, 0) \leq p \leq Ke^{-rT} \tag{2.12}$$

If the underlying asset is a stock with dividend payment, and *D* denotes the present value of the dividend payments during the life of the option. The revised bounds for European calls on stock with dividends should be:

$$S_0 - D - Ke^{-rT} \leq c \leq S_0 \tag{2.13}$$

Or

$$\max(S_0 e^{-qT} - Ke^{-rT}, 0) \leq c \leq S_0 \tag{2.14}$$

where q denotes the dividend yield that paid by the underlying stock. For short-life option, it is usual to assume the cash dividend payments are known and its present value is denoted by *D*, however, for long-life options, it is more general to assume the dividend yield are known rather than the cash dividend payment.

Similarly, the revised bounds for European put options on stock with dividend payments can be expressed in Equation (2.15) and (2.16).

$$D + Ke^{-rT} - S_0 \leq p \leq \ Ke^{-rT} \tag{2.15}$$

Or

$$\max(Ke^{-rT} - S_0 e^{-qT}, 0) \leq p \leq \ Ke^{-rT} \tag{2.16}$$

2.4. The Black-Scholes-Merton Model

The idea of Black-Scholes-Merton pricing model is to create a replicate portfolio with underlying assets and riskless assets, and adjust the position of underlying assets according to the change of the price of underlying assets. Under the assumption of the absence of arbitrage opportunity, the present value of the replicate portfolio is equal to the value of the option.

This chapter is written based on the information stated by Hull (2009), Marlow (2001), Ursone (2015) and Natenberg (2009).

2.4.1. The Stochastic Process for Stock Price

Since the value of stock option is derived from the value of its underlying stock, it is necessary to realize the evolution of stock prices in order to price options. Generally speaking, the stock prices are supposed to follow a Markov process, which is a particular kind of stochastic process with the characteristics that only the present value of a variable is valid for predicting the future value. It means the history value of the variable and the way this variable evolves from the past are both irrelevant with the prediction. This is consistent with the weak form of the market efficiency.

It is concluded that the stock prices are following a stochastic process by observing the price evolution of the stock in the real market. Thus, the option prices, which are typically influenced by its underlying stock prices, must follow a stochastic process as well.

The Itô process is a kind of particular generalized Markov stochastic process where the variable *a* and *b* respectively represents the functions of the variable *x* and time *t*. During the Itô process, the expected drift rate and variance rate will change as the time evolves. The Itô process can be expressed in Equation (2.17).

$$\mathrm{d}x = a(x,t)\mathrm{d}t + b(x,t)\mathrm{d}z \tag{2.17}$$

where dz is a Wiener process, which is a kind of specific Markov stochastic process with the change of mean is equal to 0 and its variance rate is 1.0 per year. And the change of variable z, Δz, in a small time interval Δt can be expressed in Equation (2.18).

$$\Delta \mathrm{z} = \varepsilon\sqrt{\Delta t} \tag{2.18}$$

Where ε has a standardized normal distribution $\emptyset(0,1)$.

In practice, we should notice that the expected percentage required return of the investors has no relationship with the stock price, moreover, the stock prices do exist volatility. Therefore, when deriving the stochastic process for stock prices, Hull (2009, p266) put forward the idea that " *A reasonable assumption is that the expected return is constant and the variability of the percentage return in a short period of time, Δt, is the same regardless of the stock price.*" Therefore leads to the following model:

$$\mathrm{d}S = \mu \mathrm{S}\mathrm{d}t + \sigma \mathrm{S}\mathrm{d}z \tag{2.19}$$

or

$$\frac{\mathrm{d}S}{S} = \mu\mathrm{d}t + \sigma\mathrm{d}z \tag{2.20}$$

Where the variable μ is the expected return of the stock prices and the variable σ represents the volatility of the stock prices, thus the variable σ^2 indicates the variance of the stock prices. The equation (2.20) reflects the most widely used stochastic model for stock price evolution and it can reflect the stock price behavior in the real world as well, while the expected return μ should be equal to the risk-free interest rate r in the risk-neutral world.

2.4.2. Itô's Lemma

The generalized Wiener process assumes the drift rate and variance rate are both constant. However, the price of the option should be a function of its underlying stock's price and time in the real world. And this is the reason for introducing the Itô's lemma.

Firstly suppose the variable x follows the Itô process, which means it has:

$$\mathrm{d}x = a(x,t)\mathrm{d}t + b(x,t)\mathrm{d}z \tag{2.21}$$

where dz is a Wiener process and a is the function of the variable x whereas b denotes the function of time t. Moreover, the mean change, which is known as the drift rate as well, of variable x is a and its variance rate is b^2. The Itô's lemma states that the function G of variable x and t follows:

$$\mathrm{d}G = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)\mathrm{d}t + \frac{\partial G}{\partial x}b\mathrm{d}z \tag{2.22}$$

where dz is the same Wiener process as we have introduced in equation (2.17). Therefore, the function G can be regarded as following an Itô process whose drift rate is:

$$\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2$$

And the variance rate is

$$\left(\frac{\partial G}{\partial x}\right)^2 b^2$$

In the previous chapter we have introduced the reasonable model for stock price movements, which can be expressed as:

$$\mathrm{d}S = \mu S\mathrm{d}t + \sigma S\mathrm{d}z$$

where μ and σ are both constant. According to the Itô's lemma, the function G of S and t follows the process:

$$\mathrm{d}G = \left(\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right)\mathrm{d}t + \frac{\partial G}{\partial S}\sigma S\mathrm{d}z \tag{2.23}$$

2.4.3. The Black-Scholes-Merton Differential Equation

The idea of deriving the Black-Scholes-Merton differential equation is to set up a riskless portfolio which contains a long or short position in the options and a opposite position in the underlying stock, while under the assumption of the absence of arbitrage opportunities, the return of the riskless portfolio must be equal to the risk-free interest rate, *r*.

In order to derive the Black-Scholes-Merton differential equation, Hull (2009) has concluded that the following assumptions must be held:

1. The stock price movements follow the model we constructed before, where μ and σ are both constant.
2. The short selling of the securities is permitted.
3. There are no taxes and transaction costs. Moreover, all the securities are perfectly divisible.
4. During the life of the options, the stocks do not make dividend payments.
5. There does not exist arbitrage opportunities.
6. The securities trading is a continuous process.
7. The risk-free interest rate *r* is constant, and is the same for all the maturities.

Suppose we consider the price of the option at time *t*, not time zero, which means if the expiration date of the option is *T*, then the life of the option should be *T-t*. Then we assume that the stock price movements follow the process we constructed before, which is:

$$\mathrm{d}S = \mu S\mathrm{d}t + \sigma S\mathrm{d}z$$

In a small time interval, which is denoted by Δt, the change in the stock price *S*, which is denoted by the variable ΔS, can be derived as:

$$\Delta S = \mu S\Delta t + \sigma S\Delta z \tag{2.24}$$

And this is known as the discrete version of Equation (2.19). Now suppose that *f* is the price of a call option which is derived from *S*, then the variable *f* must be the function of *S* and *t*. Therefore, according to the Itô's lemma, it must have:

$$\mathrm{d}f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\mathrm{d}t + \frac{\partial f}{\partial S}\sigma S\mathrm{d}z \tag{2.25}$$

And the discrete version of Equation (2.25) is:

$$\Delta f = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial f}{\partial S}\sigma S\Delta z \tag{2.26}$$

Where Δf is the change of f in a small time interval, Δt.

Since the f has the same Wiener process with S as we have discussed in the Itô's lemma. This means the Δz, which is equal to $\varepsilon\sqrt{\Delta t}$, must be the same in the Equation (2.24) and (2.26). Therefore, we can eliminate the Wiener process by setting the portfolio with options and its underlying stocks. The portfolio can be:

-1: derivative (option)

$+\partial f/\partial S$: shares

The expression above indicates that the holder of this portfolio has a short position in one option and a long position in the amount of $\partial f/\partial S$ of shares. Suppose the value of this portfolio is denoted by $\prod$, thus:

$$\prod = -f + \frac{\partial f}{\partial S}S \tag{2.27}$$

The change in the value of the portfolio, $\Delta\prod$, in the time interval Δt, is:

$$\Delta\prod = -\Delta f + \frac{\partial f}{\partial S}\Delta S \tag{2.28}$$

Since we know the value of the variable Δf and ΔS from the Equation (2.24) and (2.26), which can substitute in Equation (2.28), then Equation (2.28) can be revised as:

$$\Delta\prod = \left(-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t \tag{2.29}$$

By eliminating the variable Δz, the portfolio can be proved to be riskless during the time interval Δt. Under the assumption of the absence of arbitrage opportunities, which means the portfolio must earn the same instantaneously rate of return as other short-term riskless securities, which leads to:

$$\Delta\prod = r\prod\Delta t \tag{2.30}$$

After substituting from the Equation (2.27) and (2.29) into Equation (2.30), it follows:

$$\left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t \tag{2.31}$$

Thus

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 = rf \tag{2.32}$$

Equation (2.32) is the Black-Scholes-Merton differential equation. And the solutions of

Equation (2.32) can be various, depending on the boundary conditions that been used. The boundary conditions define the value of the option at the boundaries of the variable *S* and *t*.

The boundary condition for a European call option is:

$$f = \max(S - K, 0), when\ t = T$$

The boundary condition for a European put option is:

$$f = \max(K - S, 0), when\ t = T$$

2.4.4. The Black-Scholes Pricing Formulas

The most famous solution to the Black-Scholes-Merton differential equation is the Black-Scholes pricing formulas for the prices of European call options and European put options at time 0, both on a non-dividend-paying stock. And the formulas respectively are

$$c = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{2.33}$$

and

$$p = K e^{-rT} N(-d_2) - S_0 N(-d_1) \tag{2.34}$$

Where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Where variable *c* represents the price of European call option and *p* is the price of European put option, S_0 is the underlying stock price at time 0, *K* is the exercise price and *r* represents the continuously compounded risk-free interest rate, σ is the volatility, *T* is time to maturity of the option. The meaning of *N(x)* is explained by Hull (2009, p291) as: "*The function $N(x)$ is the cumulative probability distribution function for a standardized normal distribution.*" This means the value of function $N(x)$ is equal to the probability that a variable, which has a standardized normal distribution of $\emptyset(0,1)$, will be less than *x*, and this explanation can be displayed in Figure 2.1, where the shaded area represents *N(x)*.

In practice, the interest rate *r* is set to be equal to the zero-coupon risk-free interest rate for the time to maturity *T*. It means we can apply the yield of government bonds with the same time to maturity *T* to represent the value of *r*. Moreover, the time to maturity *T* is measured as trading days rather than calendar days, because the empirical researches prove that compared with period of trading closed, the volatility is much higher when the exchange-traded market is open. As a result, the life of the option is measured using trading days, as well as when estimating the historical volatility from historical data. If the life of the option is calculated as T years, then is can be obtained by

$$T = \frac{Number\ of\ trading\ days\ until\ option\ maturity}{252} \tag{2.35}$$

Figure 2.1 Explanation of *N(x)*

Source: Hull (2009, p291).

2.4.5. Pricing Formulas on Options Paying Dividends

One of the assumptions of the Black-Scholes model describes that during the life of the options, the underlying stocks do not make dividend payments. Therefore, the Black-Scholes option pricing formulas cannot be applied to calculate the prices of options on stocks paying dividends.

In order to solve this problem, assuming there exist continuous dividend payment of the underlying stock, and the dividend yield of stock is denoted by q. Then the differential equation can be revised as:

$$\frac{\partial f}{\partial t} + (r - q)S\frac{\partial f}{\partial S} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 = rf \tag{2.36}$$

Thus, the option pricing formula for options on stocks paying known continuous dividend yield can be expressed in Equation (2.37) and (2.38), which is derived by Merton in 1973 firstly.

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2) \tag{2.37}$$

and

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1) \tag{2.38}$$

Where

$$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Furthermore, if the dividend yield is known during the life of the options but it is not constant, Equation (2.37) and (2.38) are still working while the dividend yield q should be equal to the average annualized yield during the life of the option.

2.5. Lookback Options Pricing

As we introduced before, the lookback option can be classified into floating lookback with no exercise price and fixed lookback option with specific exercise price. Since the value of lookback option are highly correlate with the maximum or the minimum price of the underlying securities during the life of the option, the lowest price observed is denoted by S_{min} while the highest price observed is denoted by S_{max}. And the letter b represents the cost of rate, for example, it equals to the cost of interest plus any additional costs; in our cases, the value of b is equal to the difference between the riskless rate, r, and the dividend yield, which is denoted by the letter q.

Most of descriptions in this chapter is based on the information from Haug (2007) and Buchen (2012).

2.5.1. Floating Lookback Options

For floating lookback call options, under the condition that b is equal to zero, the pricing formula is:

$$c = Se^{-rT}N(a_1) - S_{min}e^{-rT}N(a_2) + Se^{-rT}\sigma\sqrt{T}[n(a_1) + a_1(N(a_1) - 1)] \tag{2.39}$$

Where $n(a_1)$ represents the standardized normal density function of a_1, and S denotes the current stock price. And under the condition that b is not equal to zero, the pricing formula for floating lookback call options is :

$$c = Se^{(b-r)T}N(a_1) - S_{min}e^{-rT}N(a_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{min}}\right)^{-\frac{2b}{\sigma^2}} N\left(-a_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}N(-a_1)\right] \quad (2.40)$$

Where

$$a_1 = \frac{\ln(S/S_{min}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.41)$$

$$a_2 = a_1 - \sigma\sqrt{T} \quad (2.42)$$

The Equation (2.41) and (2.42) holds for both conditions no matter the value of b is equal to zero or not.

For the floating lookback put options, we also consider conditions with different value of b. If the cost of rate is equal to zero, then

$$p = S_{max}e^{-rT}N(-b_2) - Se^{(b-r)T}N(-b_1) + Se^{-rT}\sigma\sqrt{T}[n(b_1) + b_1(N(b_1))] \quad (2.43)$$

And if b is equal to zero then the pricing formula should be

$$p = S_{max}e^{-rT}N(-b_2) - Se^{(b-r)T}N(-b_1) + Se^{-rT}\frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{max}}\right)^{-\frac{2b}{\sigma^2}} N\left(b_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}N(b_1)\right] \quad (2.44)$$

Where

$$b_1 = \frac{\ln(S/S_{max}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.45)$$

$$b_2 = b_1 - \sigma\sqrt{T} \quad (2.46)$$

2.5.2. Fixed Lookback Option

For fixed lookback option with the specific strike price, K, the pricing formula should be revised, because the comparison of the value between the strike price and the maximum or the minimum price observed during the life of the option should be taken into account.

For fixed lookback call options, when the strike price is greater than the maximum assets price observed, it holds:

$$c = Se^{(b-r)T}N(d_1) - Ke^{-rT}N(d_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(-d_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}N(-d_1)\right] \quad (2.47)$$

Where

$$d_1 = \frac{\ln(S/K) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.48)$$

$$d_2 = d_1 - \sigma\sqrt{T} \quad (2.49)$$

And when $K \leq S_{max}$, the pricing formula for fixed lookback option is:

$$c = e^{-rT}(S_{max} - K) + Se^{(b-r)T}N(e_1) - S_{\max}e^{-rT}N(e_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{max}}\right)^{-\frac{2b}{\sigma^2}} N\left(e_1 - \frac{2b}{\sigma}\sqrt{T}\right) + e^{bT}N(e_1)\right] \quad (2.50)$$

Where

$$e_1 = \frac{\ln(S/S_{max}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \quad (2.51)$$

$$e_2 = e_1 - \sigma\sqrt{T} \quad (2.52)$$

And for the fixed lookback put options, for the condition that the exercise price is lower than the minimum asset price achieved during the life of the option, the pricing formula should be described as:

$$p = Ke^{-rT}N(-d_2) - Se^{(b-r)T}N(-d_1) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(-d_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}N(-d_1)\right] \quad (2.53)$$

For the condition that $K \geq S_{min}$, the pricing formula should be revised as:

$$p = e^{-rT}(K - S_{min}) - Se^{(b-r)T}N(-f_1) + S_{min}e^{-rT}N(-f_2) + Se^{-rT}\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{min}}\right)^{-\frac{2b}{\sigma^2}} N\left(-f_1 + \frac{2b}{\sigma}\sqrt{T}\right) - e^{bT}N(-f_1)\right] \quad (2.54)$$

Where

$$f_1 = \frac{\ln(S/S_{min}) + (b + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{2.55}$$

$$f_2 = f_1 - \sigma\sqrt{T} \tag{2.56}$$

2.6. Summary

Chapter 2 is the theoretical part of the thesis, all the option prices we will calculate in the following chapters are depending on the information introduced in this chapter.

In this chapter, we have introduced the basic characteristics and the classifications of the option while laying emphasis on the stock option and the lookback option. The put-call parity, risk-neutral pricing theory and non-arbitrage pricing principle are the fundamentals of option pricing process. On the basis of these theories, we move one step further to derive stochastic process for stock price and the Itô's lemma and after that we obtain the Black-Scholes pricing formula for pricing options on stock with no dividend paying, and the pricing formula for options on paying continuous dividend. In the end of Chapter 2, we introduced the pricing formula for both fixed lookback option and floating lookback option as well.

3. The Impact of the Idea of Implied Volatility

The volatility of the underlying assets of options is a very important variable for pricing the options, because it plays a significant role on the measurement of the uncertainty of income. However, the volatility of the stock price is the only input which cannot be directly observed in the Black-Scholes pricing model. The volatility measures the level of uncertainty of the return of underlying stocks. In practice, the traders usually work with the implied volatility, which can indicate the information observed from the option price.

3.1. The Historical Volatility

In this chapter, most of the descriptions are based on the information from Natenberg (2014).

Since the volatility cannot be observed directly, the simplest way to estimate this variable is to use the historical data. The historical volatility is the volatility that directly measures the movement of the price of underlying asset based on the statistical analysis of history, while assuming the future is the extension of the past. By using the historical volatilities, we can forecast the future volatilities. This method of forecasting are known as backward looking, the principles for this method is easy to understand.

If the historical volatilities of an underlying stock over the past 10 years drops in the range from 20% to 60%, then the prediction for its future volatilities are more likely to fall in this range as well, under the assumption of the absence of any extraordinary circumstances. In order to calculate the historical volatilities, it is necessary to estimate the time interval between continuous price changes. The time intervals can be daily price changes, weekly price changes or monthly price changes. However, the different time intervals will not influence the result of volatilities greatly according to empirical facts.

It is easy to calculate the historical volatilities for underlying stocks follow three steps:

1. Collecting the prices of underlying stocks for fixed time intervals from the stock markets, for example the daily prices of the underlying stock.
2. For each time interval, calculate the natural logarithm of the ratio between the stock price at the end of the time period and the stock price at the beginning of the time period.
3. Calculate the standard deviation of the values mentioned above, then multiplies by the square root of the amount of the time periods that contains in one year, for example, if the fixed time interval is from day to day, it means there exist 252 trading days after deducting the closing days of the stock market. And the historical volatilities of the underlying stock is calculated as the standard deviation of the natural logarithm of the

ratio of stock prices multiplies the square root of the number of trading days.

Using the historical volatilities to forecast the future volatilities of the underlying stock is a widespread method, nevertheless it has several disadvantages. First of all, this method primarily based on the study of the historical data rather than the analysis of the current market situation, as a result, using the historical volatilities to forecast the future volatilities does not take factors, such as the new market information and the changes of stock market, in to consideration.

In the second place, the historical law only can be applied to forecast the future evolution of volatilities under the assumption that the history will repeat itself. For instance, the subprime crisis, which is an extreme event happened in 2008, had led to the high volatilities of the stock prices. However, this subprime crisis will not happen in the future for sure. Thus, using the historical data of 2008 to predict the volatilities of the underlying stock prices will definitely lead to inaccurate result.

3.2. The Implied Volatility

Most of the descriptions in this chapter are based on the information from Natenberg (1994) and Hull (2009).

The implied volatility is related to the historical volatility while they are distinct. The historical volatility is the volatility that correlated with the price movements of the underlying assets of options, and it focuses more on the reflection of the past and current conditions of the market. In contrast, the implied volatility is the volatility that determined by the price of option itself rather than the price of its underlying stock, and it focuses more on the reflection of the investors' expectations for the future.

The implied volatility is the volatility that must be fed into the theoretical pricing model, for example the Black-Scholes pricing model, to yield a theoretical value identical to the price of the option in the marketplace. The implied volatility is a very effectively estimation of volatility, it also can reflect the forecast of the future volatility of the underlying assets as the historical volatility.

3.2.1. The Calculation of Implied Volatility

When using the Black-Scholes pricing model as the theoretical pricing model to price the options, as we introduced in the previous chapter, it follows the Black-Scholes formula, for simplicity, if we take the options for non-dividend-paying stocks as example, then the prices of options can be calculated through Equation (2.33) and (2.34):

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

and

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

Where *c* represents the price for European call option and *p* stands for the price for European put option; S_0 is stock price; *K* represents the exercise price; *T* stands for time to maturity of the option and *r* is the risk-free interest rate; σ stands for the volatility and $N(x)$ represents the accumulative probability distribution.

The Black-Scholes pricing formula demonstrates that the option price is the function of S_0, *K*, *r*, *T* and σ. Under the circumstance that the value of S_0, *K*, *r* and *T* are known, and the corresponding option price in the marketplace can be collected as well. Thus, volatility σ becomes the only unknown variable, and can be calculated through Equation (2.33) and (2.34). The result of this calculation is the implied volatility.

However, the Black-Scholes pricing formula is so complex that it is difficult for us to calculate the accurate value of implied volatility. By using the function *Solver* in the Excel program, we can obtain the implied volatility after adequate iterations. When applying *Solver*, the target value that we aim at should be the minimal value of the square of difference between the market price of options, which can be found through the internet, and the theoretical value of options, which refers to the value that calculated through Black-Scholes model. And we can set the implied volatility as the volatile variable with the only constraints that its value cannot be negative.

The idea of implied volatility demonstrates that for each option with different exercise price and different time to maturity, there should exist different volatility respectively. However, one of the assumptions of the Black-Scholes pricing model is that the volatility is constant for all options. The inconsistency of these two ideas illustrates that the Black-Scholes pricing model has the tendency to misestimate the value of options.

For options on stocks that paying continuous dividend at dividend yield, q, as we introduced before, the BS pricing formula can be revised as shown in Equation (2.37) and (2.38). In order to calculate the value of implied volatility, there exist another unknown variable, which is the dividend yield. In order to estimate the dividend yield first, the put-call parity can be applied because the principle of put-call parity works under any circumstance regardless of what kind of volatility been considered.

The put-call parity for options on stock paying dividend yield at q is shown in Equation (2.8) as following:

$$c + Ke^{-rT} = p + S_0 e^{-qT}$$

From Equation (2.8), we can backward derive the value of dividend yield q, which can be expressed through Equation (3.1):

$$q = -\frac{1}{T}\ln\frac{c - p + Ke^{-rT}}{S_0} \tag{3.1}$$

The dividend yield that can be derived backward following Equation (3.1) is known as implied dividend yield. As a result, for each time to maturity and for each strike price, there will be different dividend yield applied. And for particular exercise price and maturity date, the estimation of dividend yield q can be unreliable, but based on a huge amount of matched pairs of call and put options, the estimation of dividend yield can be clear. Alternatively, we can use the function *Solver* in the Excel to calculate the dividend yield by setting the difference between $c + Ke^{-rT}$ and $p + S_0 e^{-qT}$ as the target value and set the target value equal to zero. The constraint is the range of dividend yield, which should be fell into the range of $[0,1]$.

In addition, for options with the same time to maturity, even the exercise price differs, but the dividend yields for them are not much different. Therefore, in the thesis, we only consider the condition that dividend yield q differs from time to time, which means for options with different exercise prices but the same maturity date, we apply the same value of q. And for simplicity, we set the expectation (average value) of all dividend yields with the same maturity date as the estimated dividend yield. Furthermore, when introduced Merton's pricing formula for options on stock paying dividend, we have discussed that if the dividend yield is known during the life of the options but it is not constant, Equation (2.37) and (2.38) are still working. And the solution is to set the dividend yield q equal to the average annualized yield during the life of the option.

3.2.2. Volatility Index

The Chicago Board Options Exchange (CBOE) had introduced an index to reflect the expectation of the volatility in the market for the following thirty days since 1993. The implication of this index is clearly presented on the website of Chicago Board Options Exchange as:

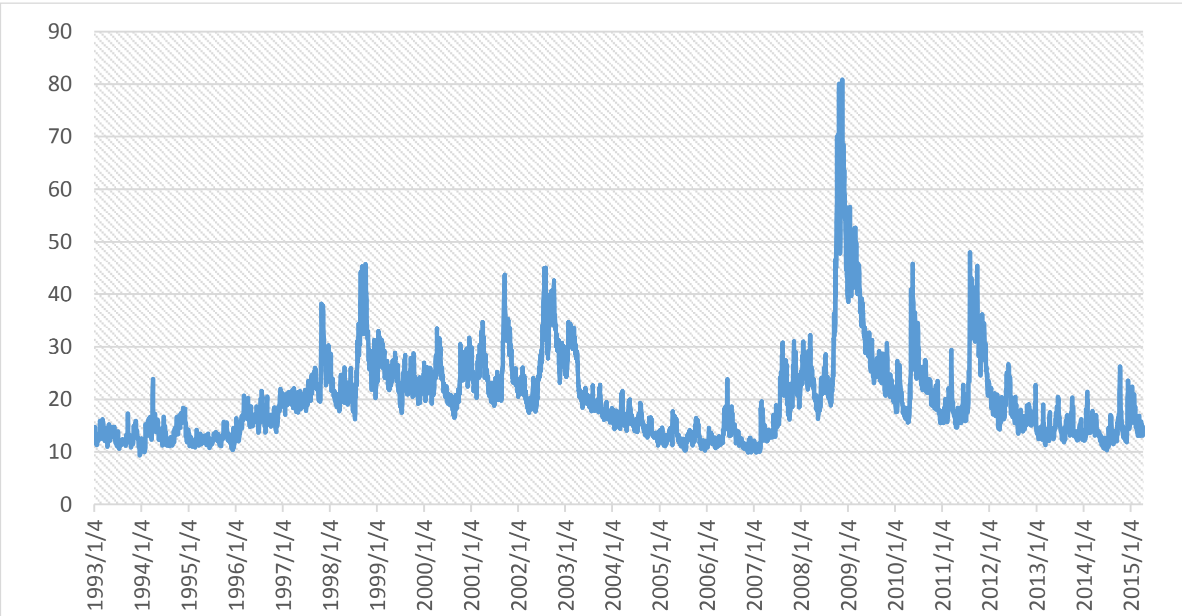
"The CBOE Volatility Index (VIX) is a key measure of market expectations of near-term volatility conveyed by S&P 500 stock index option price, it is the square root of the annualized forward price of the thirty days variance of the S&P 500 return based on a replicating portfolio of options delta-hedged with stock index futures."

In other words, the VIX is based on the option pricing process and it is used to measure perceived market risk and uncertainty.

There are also similar index such as the CBOE NASDAQ-100 Volatility Index (VXN), which conveyed by NASDAQ-100 index option prices; and the CBOE DJIA Volatility Index (VXD), which is based on the real-time prices of options on the Dow Jones Industrial Average.

The VIX reflects the "fear gauge" of the investors, the higher the VIX, the more tension and "fearful" the investors will feel about the stock market. For instance, most of the investors were fearful about the market throughout the end of 2008 to the beginning of 2009, which is the period of occurrence of the subprime mortgage crisis.

Figure 3.1 The VIX Evolution from 1993 to 2015



Source: data from Chicago Board Options Exchange with own arrangement.

The evolution of the VIX from 1993 to 2003 is displayed in Figure 3.1, and it demonstrates the VIX raised sharply from the end of 2008, and reached the peak in the beginning of 2009, which is in accordance with the appearance of the global financial crisis.

However, the VIX then began to decline in 2009 while the market continued to decline and the financial crisis continued at the same period. In fact, because VIX not only measures the fear of the investors but measures the uncertainty of the market as well, despite the market continued to decline in the 2009 but some of the market uncertainty had eliminated because of some government plans such as the Trouble Asset Relief Program (TRAP). As a result, the investors become more confident about the market than before and they were not willing to pay such high prices for the options as a protection, the decline of the prices of options lead to the decreasing level of VIX.

The VIX is calculated daily and it is an annual percentage, which means it should divide by square root of 12 in order to realize the real meaning of this index. For instance, if today's VIX is 17.03, and $17.03/\sqrt{12} = 4.92$, which means the S&P 500 is expected to fluctuate by 4.92% in the following thirty days.

VIX is a measurement of implied volatility, which is based on the prices of options, and one of the characteristics of the option prices is that they tend to increase during the turbulent period and decline when the uncertainty of market dissipate.

Figure 3.2 Daily Closing Prices for The S&P 500 And VIX During The Third Quarter of 2012

Source: Bloomberg.

For instance, the Figure 3.2 shows the relationship between the SPX and the VIX index during the third quarter of 2012. From Figure 3.2 we can state that the VIX index has fluctuated in the opposite direction of the S&P 500 index in the most of time. Under the normal circumstance, these two indexes typically move inversely. when the SPX goes up, the VIX index tends to go down, because the investors are optimistic about the market, they expect the future volatility of the market will be very low; and when the SPX goes down, the VIX index tends to increase, which reflects the investors' non-confidence towards the market, they expect the future volatility of the market will be high.

3.3. The Volatility Smile

In this chapter, most of descriptions are based on the information from Hull (2009) and Derman and Kani (1994).

One of the most important assumptions of the Black-Scholes option pricing formula is that the volatility of the underlying assets is constant, however, this is also the biggest disadvantage of this model. Since the volatility cannot be observed conveniently and directly, we can work backward from the option pricing formula to calculate the implied volatility of the options when the option prices are known. However, the condition in real world is different from the assumptions of BS model, which means the implied volatilities are different for options with different strike prices and different maturities.

A volatility smile refers to the plot of the implied volatility of an option as ordinate axis and its strike price on the horizontal axis. The volatility smile describes the law that the implied volatility for options with the same maturity date changes as the strike price of the option changes. This kind of diagram is called volatility smile since the implied volatilities of out-of-the-money options and in-the-money options are higher than the implied volatility of at-the-money option, which turns the diagram to the U-shaped curve and looks like a smile. This shape also reflects the implied volatilities are not the same for in-the-money, out-of-the-money and at-the-money options, even holding all other conditions the same.

The implied volatility is an expression of the option prices in nature, and the existence of volatility smile indicates that the Black-Scholes pricing model has the tendency to underestimate the value of deep in-the-money options and deep out-of-the-money options.

3.3.1. Volatility Smile for Calls and Puts

For European call and put stock options with the same strike price and time to maturity, if we assume the underlying asset pays dividend at yield of q, according to the put-call parity introduced before, the relationship can be expressed as follow.

$$\mathrm{p} + S_0 e^{-qT} = c + Ke^{-rT}$$

The put-call parity relationship of European options is simply based on the non-arbitrage argument, which means the establishment of this relationship does not need take the type of probability distribution of the underlying asset price into consideration. This relationship is both true under the condition that the asset price follows lognormal distribution or does not follow lognormal distribution.

By applying the Black-Scholes model, with particular value of volatility, we can calculate the theoretical prices of European call option and put option, which can be denoted by p_{BS} and c_{BS}. Since it holds put-call parity, the relationship is described as follow:

$$p_{BS} + S_0 e^{-qT} = c_{BS} + Ke^{-rT} \tag{3.2}$$

If the market prices of these options are denoted by p_{mkt} and c_{mkt}, the put-call parity also holds for the market values in the absence of arbitrage opportunities. Therefore, the relationship is

$$p_{mkt} + S_0 e^{-qT} = c_{mkt} + Ke^{-rT} \tag{3.3}$$

Simultaneous two equations above, we get

$$p_{BS} - p_{mkt} = c_{BS} - c_{mkt} \tag{3.4}$$

Equation (3.4) proves that the dollar pricing error for European put option and European call option with the same exercise price and time to maturity, must be the same when applying the Black-Scholes model to price these options.

Suppose for the particular implied volatility of a put option, i.e. 15%, which indicates that when applying a volatility of 15% in the Black-Scholes model to price put option, there exist p_{BS} is equal to p_{mkt}. Moreover, from equation (3.4) it holds c_{BS} is also equal to c_{mkt} when apply the volatility of 15%, hence, the implied volatility for call option with the same time to maturity and exercise price is 15% as well.

Therefore, according to the argument above, the implied volatility of a European call option is always equal to the implied volatility of a European put option with the same exercise price and time to maturity. This indicates that the volatility smile for European call option is also the same with the volatility smile for European put option when the options have the same strike price and time to maturity.

3.3.2. The Cause of the Volatility Smile

One of the explanation of the existence of the volatility smile is the difference between the distribution of the market and the distribution of the Black-Scholes pricing model.

The Black-Scholes pricing model assumes that the price of underlying assets follow the lognormal distribution and the yield follows the normal distribution. When consider an asset price follows a lognormal distribution, two basic conditions must be satisfied: firstly, the volatility of this asset must be constant; for the second point, the price must changes smoothly, which means there should be no jumps in the price change. However, in real world, a large number of empiricism prove that the distribution of yield is more likely to have the characteristic of “fat tail” compared with lognormal distribution. This characteristic describes the probability that the yield can be extreme values, is higher than the normal distribution. Therefore, if we apply the assumption that yield follows the normal distribution to calculate the option price, it means the model has underestimated the probability of very high and very low yield, which simultaneously underestimate the option price for deep in-the-money option and deep out-of-the-money option.

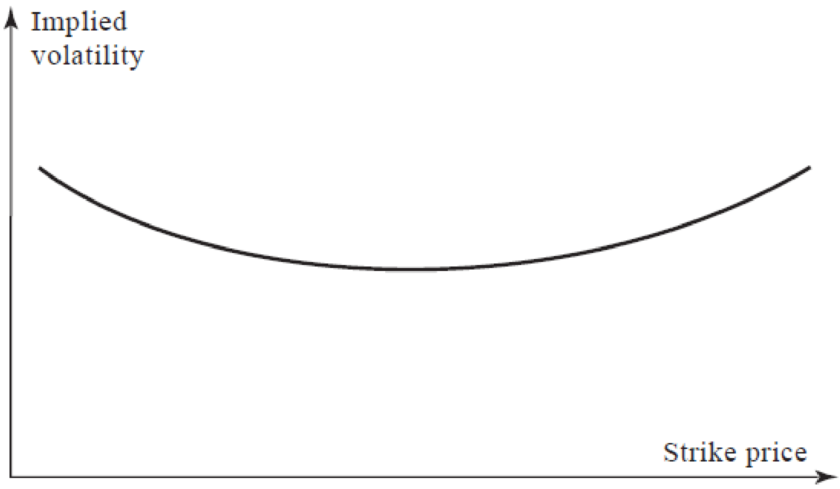
In general, the existence of volatility smile can be caused or influenced by the expectation of the future price of underlying assets as well. Assuming the current price of underlying stock is S_0, and the price is expected to fall to S_1. At this time, for put options with any given exercise price, the option prices are going to increase because the put option itself becomes more in the money; and at the same time, for all the call options with any given exercise price, the option prices are going to decrease because the call option itself becomes more out of the money. Among all the options, for out-of-the-money put options with strike prices that fall in the range from S_1 to S_0 will turn into the in-the-money put options; for these options, their prices will increase the most. Similarly, for in-the-money call options with strike prices falls in the range from S_1 to S_0 will turn into the out-of-the-money call options; and for these options, their prices will decrease the most.

As a result, under the condition that the current price of the underlying assets has not been changed, the increasing range for those out-of-the-money put options with exercise price falls in the range from S_1 to S_0, is larger than in-the-money put options. Similarly, the decreasing range for in-the-money call options with exercise price falls in the range from S_1 to S_0, is larger than out-of-the-money call options. Thus, in this condition, the volatility smile behaves as the left part overtops the right part, which is called volatility smirk.

3.3.3. The Volatility Smile for Foreign Currency Options

Generally speaking, the shape of volatility smile for foreign currency options is shown in Figure 3.3. From Figure 3.3 we can state that the volatility smile for foreign currency options is a U-shaped curve with the lowest implied volatility exists for at-the-money option, and the implied volatility will become higher as the options become more in the money or out of the money.

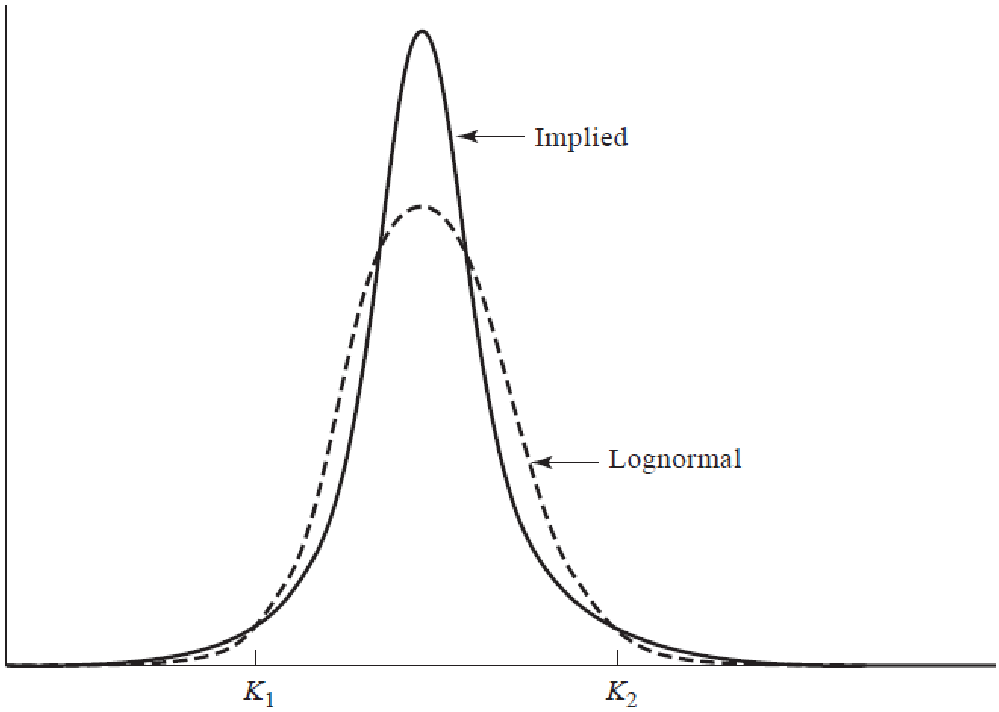
Figure 3.3 Volatility Smile for Foreign Currency Options



Source: Hull (2009, p383).

The existence of volatility smile has proved that price of underlying asset does not follow lognormal distribution, and from the curve we can derive what kind of implied distribution it should follow.

Figure 3.4 Implied Distribution And Lognormal Distribution for Foreign Currency Options



Source: Hull (2009, p383).

The implied distribution is defined as the risk-neutral probability distribution for the underlying asset price at a specific future time from the volatility smile given by options maturing at that time. And the implied distribution of foreign currency options is displayed in Figure 3.4, as well as the lognormal distribution so that a clearly difference can be illustrated.

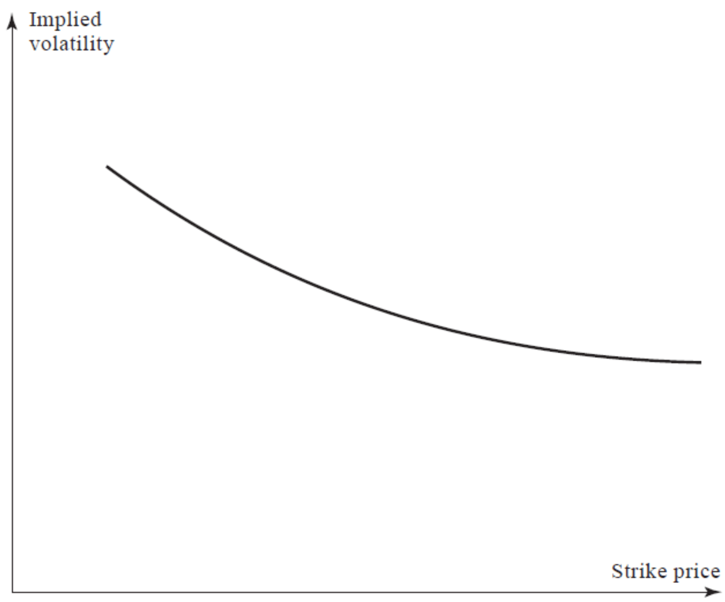
Considering a deep out-of-the-money put option with low strike price, K_1, this option will be exercised only when the exchange rate is lower than K_1. From Figure 3.4 we can state that the probability of this condition is higher for implied distribution compared with lognormal distribution. Then considering a deep out-of-the-money call option with high strike price, K_2, this option will be exercised only when the exchange rate is higher than K_2. It is shown from Figure 3.4 that the probability of this condition is also higher for implied distribution compared with lognormal distribution.

Then, a relatively higher price of the option is expected when following the implied distribution, compared with the condition that following the lognormal distribution; because the option tend to have a higher possibility to be exercised when following implied distribution. As a result, it will lead to a relatively higher implied volatility when the exchange rate become either high or low.

3.3.4. The Volatility Smile for Equity Option

Compared with foreign currency option, a large amount of empirical results prove that the volatility smile of equity option has a different shape, which is described in Figure 3.5. This kind of shaped curve is referred to as the volatility skew.

Figure 3.5 Volatility smile for equity option

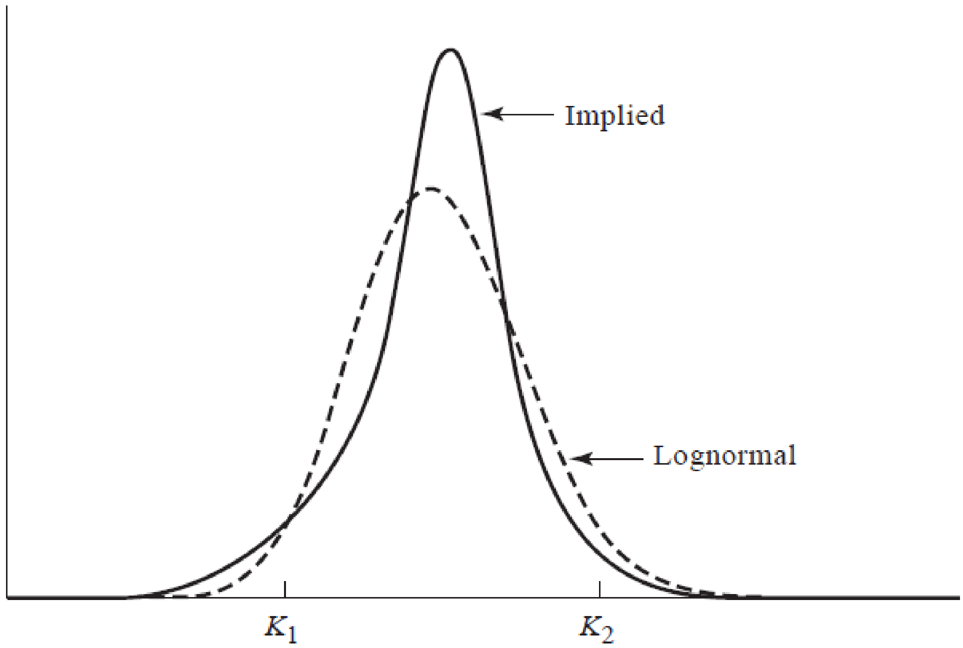


Source: Hull (2009, p386).

As the strike price increases, the implied volatility of equity option will decreases. It means for deep out-of-the-money put option and deep in-the-money call option with low strike price, the implied volatility applied to price the equity option is higher than deep in-the-money put option and deep out-of-the-money call option with high strike price. However, this is not always the truth, in real world, the volatility smile for stock options sometimes behaves similar with the currency options, which means it looks also like a “smile” rather than a skew.

The difference between volatility smile of foreign currency option and equity option is caused by the difference between the implied distributions that options followed. The implied distribution of price for equity options is shown in Figure 3.6.

Figure 3.6 Implied distribution and lognormal distribution for equity options



Source: Hull (2009, p386).

Considering a deep out-of-the-money put option with low strike price of K_1, and the probability that this option will be exercised is equal to the probability that the price is lower than K_1. And this probability is higher when the price follows implied distribution, which is illustrated in Figure 3.6. Then considering a deep out-of-the-money call option with high strike price of K_2, this option will be exercised only when the stock price exceeds K_2. From Figure 3.6 we can state that the probability of this condition is lower when the price of underlying stock follows the implied distribution compared with the lognormal distribution.

As a result, for the equity options, a higher strike price will lead to a relatively lower implied volatility while a lower strike price will lead to a relatively higher implied volatility, which shaped the curve of volatility smile as a volatility skew.

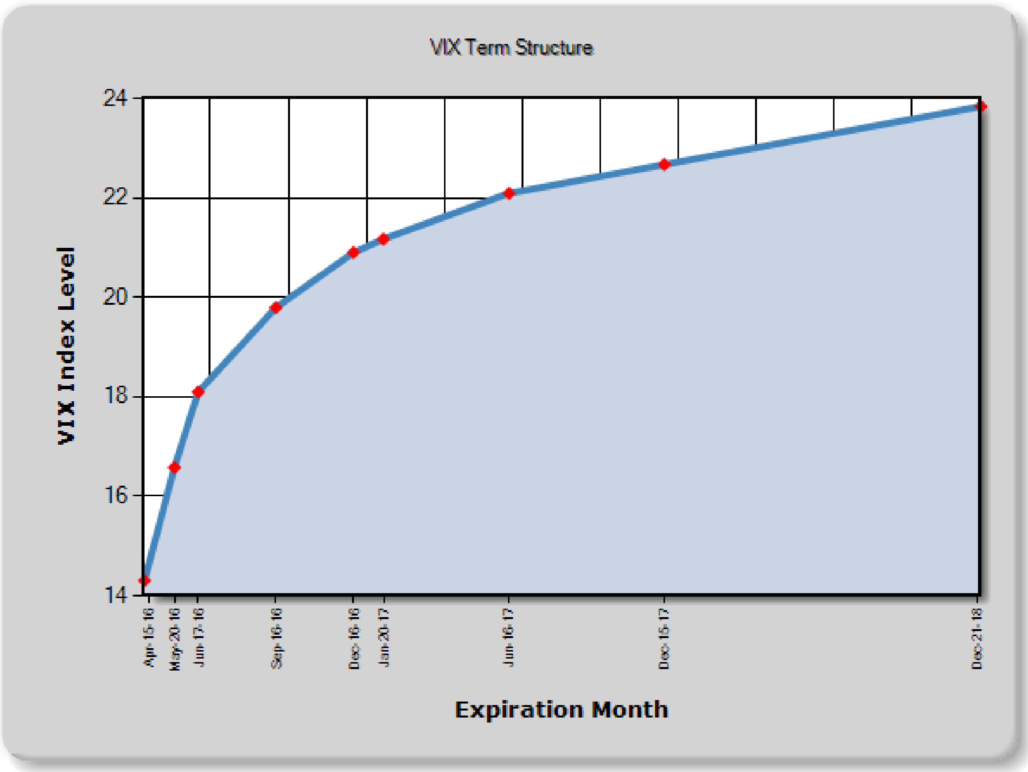
For the existence of volatility skew of equity options, one of the explanations is that it concerns leverage. When the value of a company's equity decrease, the stock price will decline and the leverage of company will increase, which means the volatility will also increase and the equity becomes more risky. In reverse, when the value of a company's equity increase, which lead the stock price increase as well, this means the leverage of company will decrease and the volatility will decrease as well, the equity will be less risky. This conclusion is consistent with Figure 3.5.

3.4. The Volatility Surface

In this chapter, most of descriptions are based on the information from Hull (2009) and Cont and Fonseca (2002).

The implied volatility differs not only because of the difference in the exercise price, but also the difference in the maturity date, or it can be expressed as time to maturity. The concept of term structure of volatility describes how the implied volatility differs for options with same underlying asset and exercise price but different maturity dates. The term structure of volatility is mostly caused by the market's implied impact of upcoming event.

Figure 3.7 VIX Term Structure



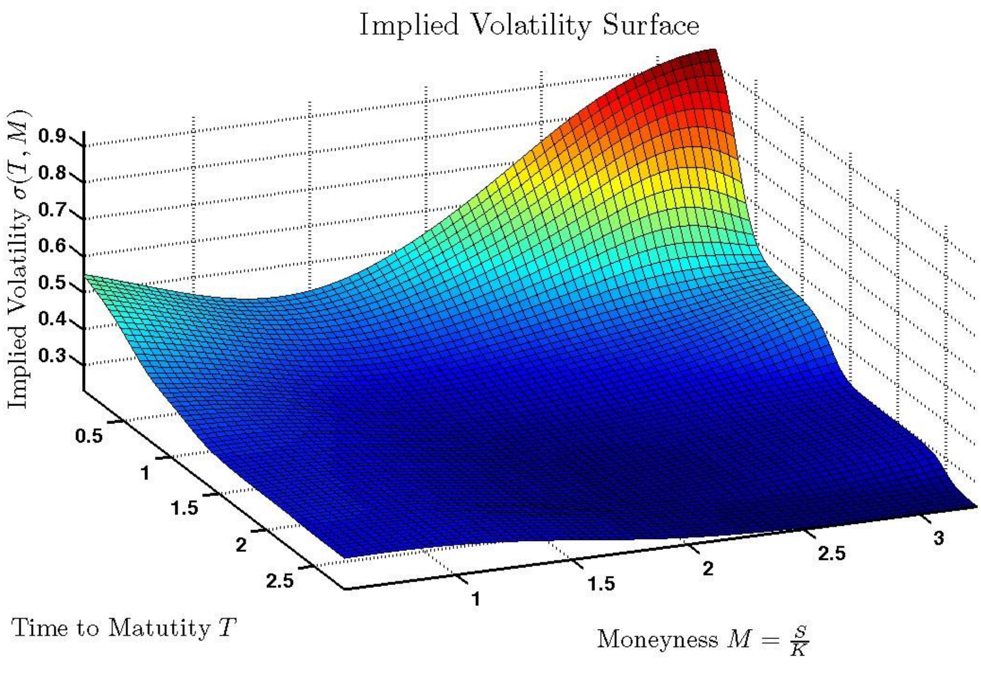
Source: Chicago Board Options Exchange.

The term structure for VIX index is displayed in Figure 3.7 as an example. By analyzing the trend of term structures of implied volatility, the investors can conclude a better expectation of the option prices because the curve of term structure suggests the market's expectation on the future volatility. For instance, the term structure for VIX index is a rising curve as shown in Figure 3.7, which means the implied volatility for long term options are higher than the implied volatility for short term options; this indicates that the short term implied volatility is expected to rise and the prices of short term options are also expected to rise.

In order to make the combination of both characteristics of implied volatility, a three dimensional surface can be constructed, which plots simultaneously the volatility smile and term structure of implied volatility together for all options on a specified underlying asset; this kind of 3D plot is known as the implied volatility surface.

For constructing an implied volatility surface, we need to collect the implied volatility of all options with same underlying asset but different exercise price and time to maturity; then we need to construct an implied volatility matrix firstly. In this volatility matrix, we set the value of time to maturity, *T*, as the first row, and set the value of K/S_0, which is known as moneyness, as the first column. And the rest of empty position is filled with the corresponding value of implied volatility. And the value of implied volatility is derived from the mid-value of corresponding implied volatility of calls and puts with same exercise price and expiration date.

Figure 3.8 Implied Volatility Surface



Source: Volatility surface image in MATLAB center.

After constructing the implied volatility matrix, we can move one step further to derive the implied volatility surface. By setting the value of implied volatility as Z-axis in general, while the time to maturity is set as Y-axis and the moneyness as X-axis, we can construct the implied volatility surface. A typical implied surface is shown in Figure 3.8.

One of the most useful application of the volatility surface is to determine an appropriate volatility to substitute into Black-Scholes-Merton pricing model in order to price stock options.

3.5. Summary

The theory of implied volatility is introduced in Chapter 3, this theory is a complement of Black-Scholes-Merton pricing model, which has the disadvantages in the assumption of the constant volatility. The option prices that calculated using constant volatility will have the tendency that underestimates the value of deep in-the-money options and deep out-of-the-money options.

The significant points in the theory of implied volatility include the comparison between historical volatility and implied volatility, the calculation of implied volatility, the CBOE volatility index and the volatility smile. Through introducing the cause of implied volatility smile and the volatility smile for foreign currency options and equity options, we can generate a comprehensive understanding of volatility smile. In the end of this chapter, we presented the term structure of implied volatility and the way to construct the implied volatility surface, which connect the volatility smile and the term structure of volatility together.

4. Comparative Analysis Using Real Market Data

According to the theory of option pricing and the idea of implied volatility we introduced in the previous chapters, we will use real market data to price both stock option and lookback option by applying the Black-Scholes pricing model. And we will also test the idea of implied volatility smile and implied volatility surface, and make the comparative analysis of option prices, which are calculated by using historical volatility, at-the-money volatility and implied volatility respectively.

4.1. Data Description

In order to price the options, the first step is to collect necessary data from the market. For accomplishing the purpose of the thesis, we choose the prices of options on stock of Apple Inc. as the real market database.

Apple Inc. is a hi-tech company that established in 1976 in the United States and designs, manufactures the mobile communications and media devices, personal computers and portable digital music players. Apple Inc. has listed on National Association of Securities Dealers Automated Quotations (NASDAQ) in 1976 with the stock code "AAPL".

One of the objective of the thesis is to use the market data to calculate the implied volatility of specific options and derive its volatility smile. Therefore, we collect the prices for the options of Apple Inc. as market data, some of the important data that must be needed according to the Black-Scholes pricing formula have been listed in the Table 4.1.

Table 4.1 Pricing Factors

Current stock price	**100.53 USD**								
Pricing date	2016/3/1								
Expiration date	2016						2017		2018
	3/18	4/15	5/20	6/17	7/15	10/21	1/20	6/16	1/19
Time to Maturity	17	44	79	106	134	230	319	465	678
T (p.a.)	0.067	0.175	0.313	0.421	0.532	0.913	1.266	1.845	2.690
Risk-free rate	0.08%	0.10%	0.17%	0.26%	0.33%	0.47%	0.60%	0.80%	1.02%
Dividend yield (BID)	3.04%	1.27%	0.85%	0.80%	0.85%	1.03%	1.54%	1.44%	1.58%
Dividend yield (ASK)	3.51%	0.96%	1.23%	1.25%	0.96%	1.23%	1.59%	1.31%	1.53%

Source: CBOE and own calculation.

The calculation of dividend yield and time to maturity, *T*, has been introduced in the previous chapters, moreover, for the risk-free interest rate, we collect the yield of U.S government bonds as the risk-free interest rate and transfer it into specified value for each time to maturity by using linear regression model. The original value of the yield of U.S government bonds is presented in Table 4.2.

Table 4.2 Yield of U.S Government Bonds

	1month	**3 month**	**6month**	**1year**	**2year**	**5year**	**10year**
riskless rate	0.08%	0.12%	0.32%	0.50%	0.86%	1.68%	2.29%

Source: Bloomberg.

Furthermore, some of the market prices for AAPL options with expiration date on March 18, 2016, are presented in Table 4.3 as an example, which includes both bid prices and ask prices for call options and put options as well as its trading volumes, and the pricing date for these market prices is on March 1, 2016. The unit for prices is USD. The phenomenon that as strike prices increase, prices for call options are decreasing is illustrated by data in Table 4.3; this is because call options tend to be more out of the money as its exercise prices increase. And it works in the opposite way for put options because puts tend to be more in the money as strike prices increase, and their price evolutions are in the same direction with its strike prices.

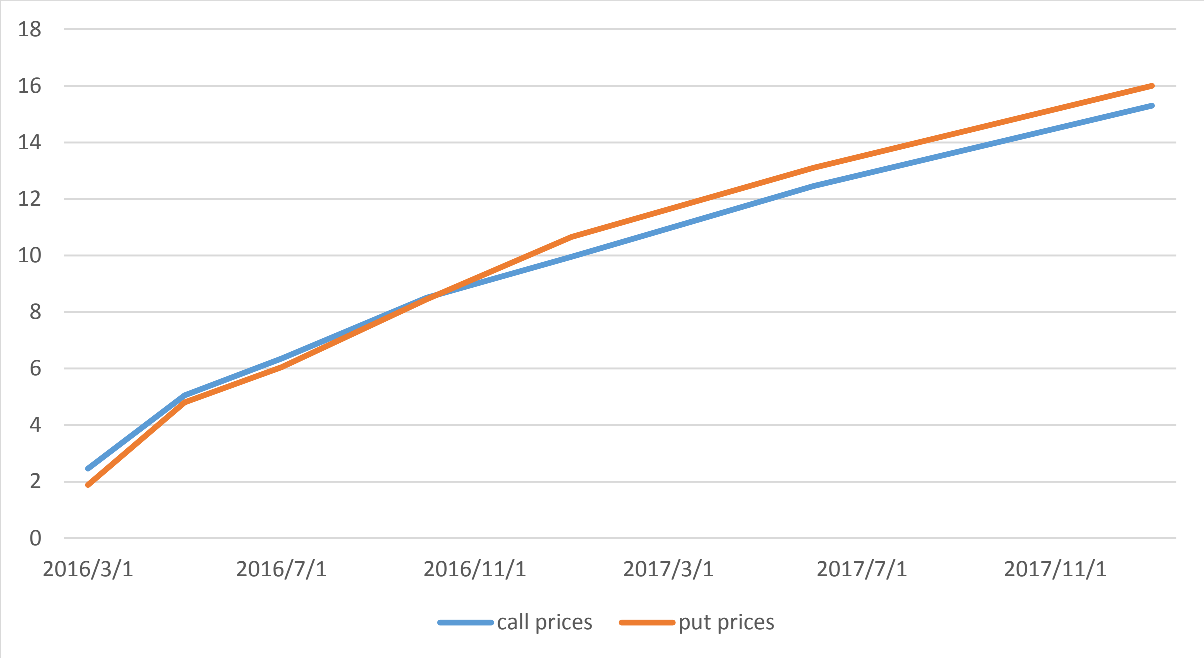
Table 4.3 AAPL Market Option Prices

Expiration Date: March 18, 2016						
Trading Volume	Bid	Ask	**Strike**	Bid	Ask	Trading Volume
368	3.7	3.8	**98**	1.19	1.24	3081
127	3.3	3.45	**98.5**	1.35	1.61	231
419	3.05	3.15	**99**	1.51	1.56	1316
294	2.68	2.75	**99.5**	1.69	1.75	1205
9058	2.46	2.55	**100**	1.88	1.95	3288
			100.53			
2989	1.95	2	**101**	2.36	2.42	2574
21240	1.5	1.54	**102**	2.88	2.97	1386
10711	1.11	1.15	**103**	3.5	3.6	212
8381	0.78	0.81	**104**	4.2	4.3	71
4174	0.55	0.59	**105**	4.95	5.1	204

Source: Chicago Board Options Exchange.

And the term structure of the market prices for AAPL stock options with exercise price equal to 100 USD, is displayed in Figure 4.1. From Figure 4.1 we can state that for both calls and puts with the same exercise price but various expiration dates, as the value of time to maturity increases, its market price will also increase on account of the increase of its time value. This principle works for all AAPL stock options regardless of its exercise prices. Thus, this can be one of examine standards for pricing AAPL stock options with different expiration date.

Figure 4.1 Term structure of AAPL stock option market prices



Source: Chicago Board Options Exchange with own arrangement.

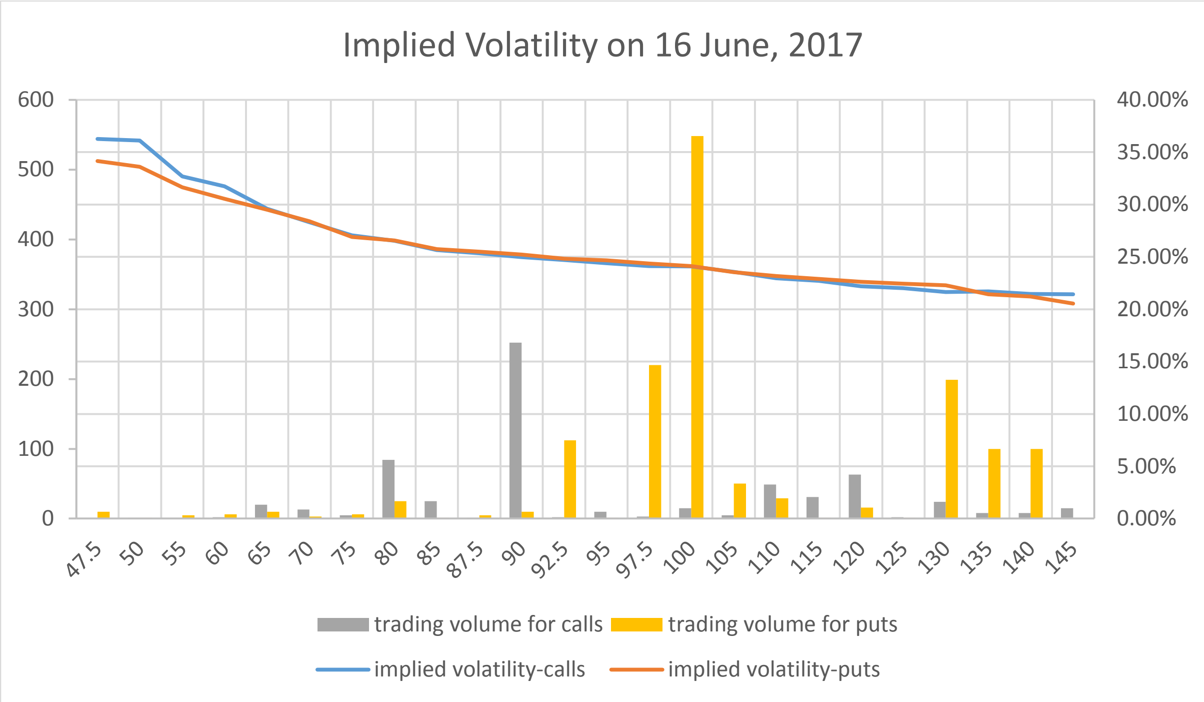
4.2. Comparative Analysis of Implied Volatility

Using the data that we have collected and then applying the Black-Scholes pricing model, we can get the results of the implied volatilities. Thus, one of the objectives of the thesis can be accomplished, which is to use the market data to calculate the implied volatility and prove the existence of the volatility smile and the term structure of the implied volatility.

Firstly, the market prices that we have collected include both bid prices and ask prices for call options and put options, thus, we can calculate the implied volatility for calls and puts respectively according to its bid and ask price. According to the theory that we introduced in the previous chapters, we can conclude that the value of implied volatility for both call options and put options with the same expiration date and exercise price should be the same according to the put-call parity. And the behavior of implied volatility calculated based on the real market data is presented in Figure 4.2, we take the results of implied volatility for AAPL stock options with expiration date on 16 June, 2017, which is calculated using the bid prices, as an example.

From Figure 4.2 we can state that the shape of implied volatility smile of AAPL stock option behaves as a volatility skew, which means a higher strike price will lead to a relatively lower implied volatility while a lower strike price will lead to a relatively higher implied volatility.

Figure 4.2 Implied Volatility for Calls and Puts with Bid Prices on 16 June, 2017



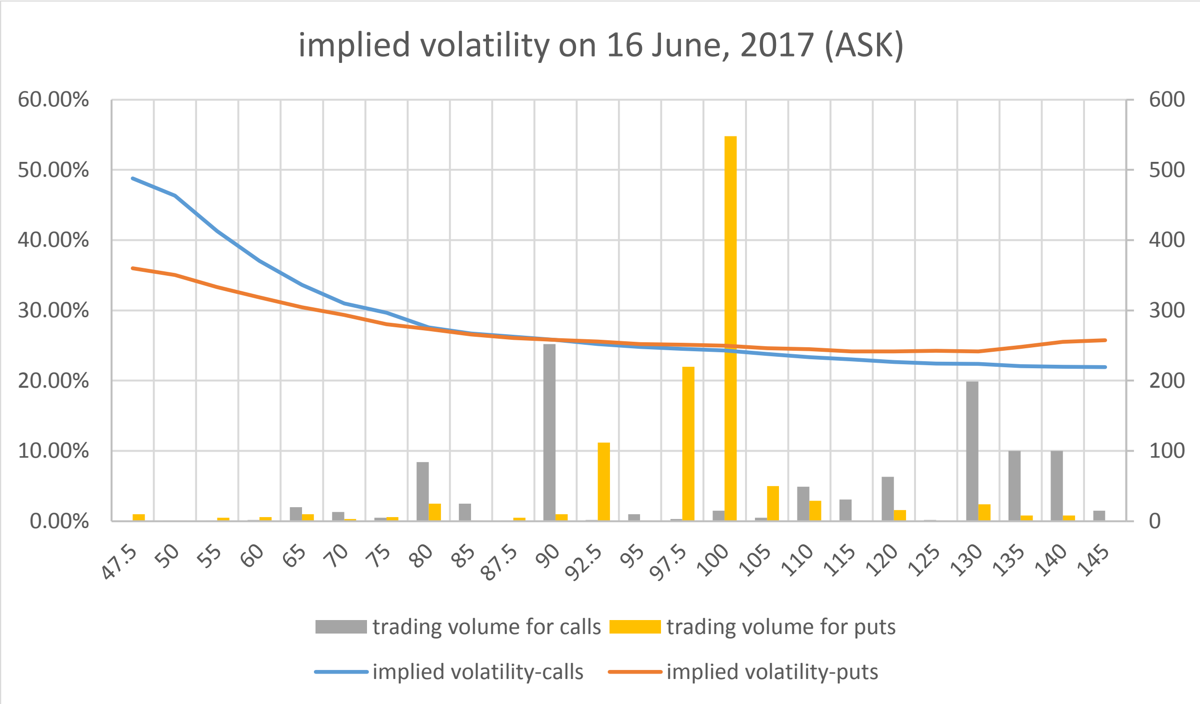
Source: Own calculation.

Moreover, the volatility smile of call options are almost coincident with the volatility smile of put options with only little bias in some exercise prices. And this result is consistent with the theory that the volatility smile for European call option is the same as for European put option when they have the same strike price and time to maturity. And the situation for AAPL stock options with the same maturity date but is calculated using ask prices can be displayed in Figure 4.3.

Seen from Figure 4.3, the implied volatility smiles for both call options and put options also shaped like volatility skews. However, under this condition, the situation for call options is little different from the situation for put options, while the range of implied volatility for call options is broader than the range for put options. This is because in the real world, when pricing the options, we need to take many other factors into consideration, for example the trading volume and the open interest.

As shown in Figure 4.3, for example, for exercise prices which are greater than 110 USD, the implied volatilities for calls are lower than the implied volatilities for puts, and at the same time, the trading volumes for call options are greater than for put options. Generally speaking, for option with higher trading volume, its implied volatility tends to be lower, which reflects the confidence of traders as well, and the results that displayed in Figure 4.2 is coincident with this empiricism.

Figure 4.3 Implied Volatility for Calls and Puts with Ask Prices on 16 June, 2017



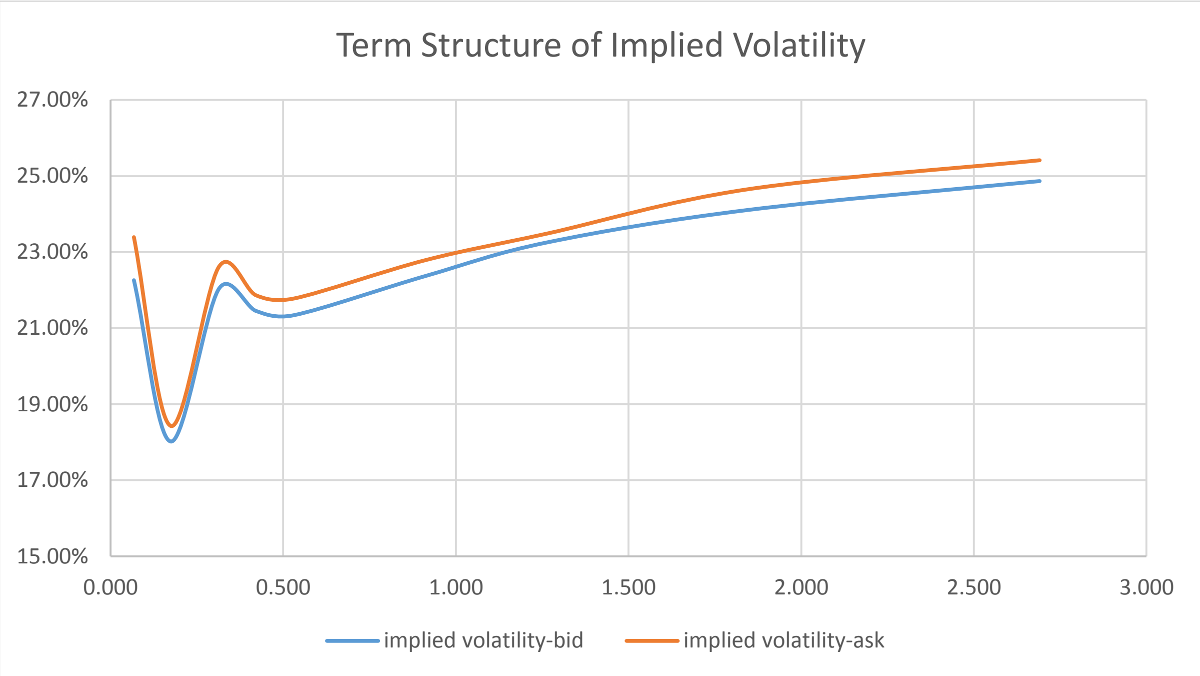
Source: Own calculation.

Another characteristic of implied volatility is its term structure, the term structure of implied volatility can be different according to different characteristics of various underlying assets. In order to examine the term structure of implied volatility, we choose the implied volatility which is calculated using both bid prices and ask prices of AAPL stock options with the exercise price of 100 USD, and maturity dates vary from March, 2016 to January, 2018. The result of the term structure of implied volatility is displayed in Figure 4.4.

From Figure 4.4 we can state that the implied volatility curves which are calculated using both bid prices and ask prices behave almost the same. However, implied volatilities for deep in-the-money options and deep out-of-the-money options is not as much accurate and reliable as the implied volatility for at-the-money options according to empirical results. Because the market prices of deep in-the-money options and deep out-of-the-money options can be influenced by many other market factors.

Therefore, the term structure of implied volatility for AAPL stock option is observed from prices of the options, which are near at the money, for the consideration of accuracy, and it is shown as a rising curve substantially, which means the implied volatilities for long term options are higher than the implied volatilities for short term options; this kind of rising curve indicates that the short term implied volatility is expected to rise and the prices of short term options are also expected to rise. This law can be later applied to examine the prices calculated for AAPL stock option.

Figure 4.4 Term Structure of Implied Volatility



Source: Own calculation.

After obtaining the implied volatility for AAPL stock options, we can move one step further to construct its volatility surface in order to price the options. Under the purpose of deriving the implied volatility surface, we need to construct an implied volatility matrix firstly. As we introduced in the previous chapter, in this volatility matrix, we set the value of time to maturity, *T*, as the first row, and set the value of K/S_0, which is named as moneyness, as the first column. And the rest of empty positions are filled with the corresponding value of implied volatilities. The value of implied volatilities are derived from the mid-value of corresponding implied volatilities of calls and puts with same exercise price and expiration date.

Table 4.4 Implied Volatility Matrix for Bid Price

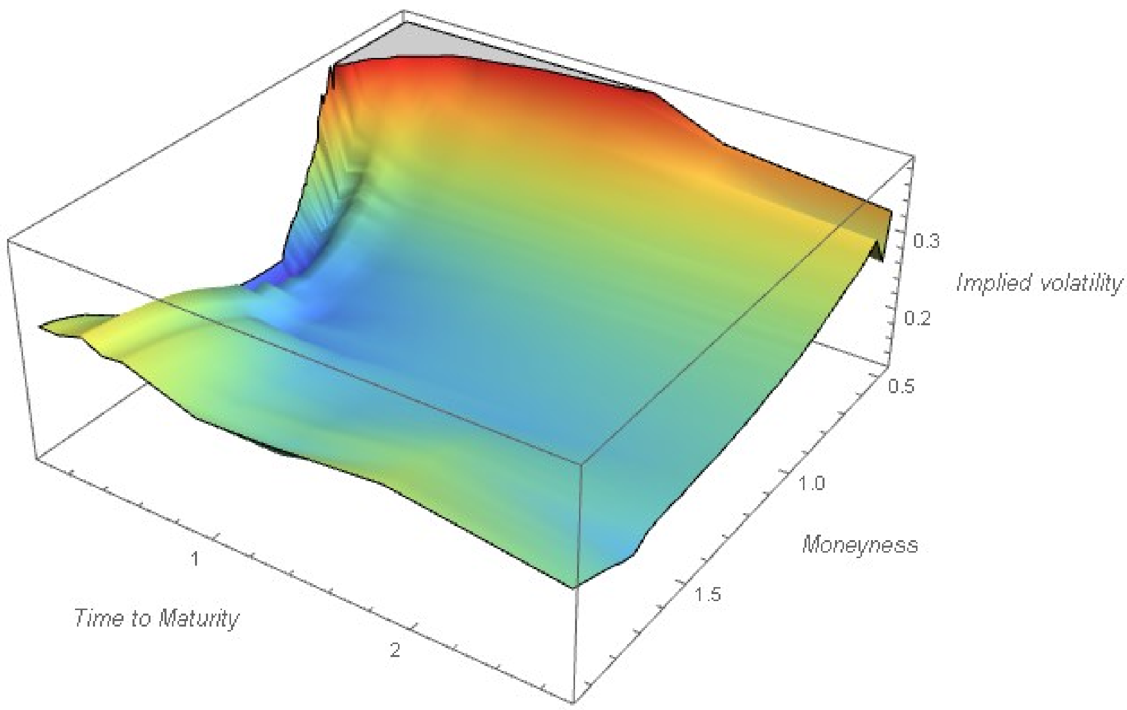
K/S_0 \ T	17	44	79	106	134	230	319	465	678
	0.067	0.175	0.313	0.421	0.532	0.913	1.266	1.845	2.690
0.990	22.27%	18.21%	22.16%	21.58%	21.43%	22.45%	23.31%	24.13%	24.88%
0.995	22.26%	18.02%	22.03%	21.45%	21.34%	22.37%	23.26%	24.11%	24.87%
1.000	22.05%	17.20%	21.93%	21.37%	21.23%	22.30%	23.19%	24.05%	24.81%
1.005	21.84%	16.38%	21.82%	21.29%	21.12%	22.23%	23.13%	23.99%	24.76%
1.015	21.18%	16.12%	21.61%	21.13%	20.89%	22.09%	23.01%	23.87%	24.66%
1.025	20.54%	15.63%	21.41%	20.97%	20.67%	21.95%	22.89%	23.76%	24.55%

Source: Own calculation.

The implied volatility matrix which filled with implied volatilities that calculated using bid prices is shown in Table 4.4. Using the implied volatility matrix, we can determine the value of implied volatility for pricing options conveniently, for example, in order to price the value of an option with the expiration date on April 15, 2016, and the exercise price is 100 USD, while the pricing date is March 12, 2016 and current stock price is 101 USD, then the value of implied volatility should be some interpolations between 18.21% and 22.27%.

The implied volatility surface constructed by implied volatilities, which is calculated using bid prices, is shown in Figure 4.5. We take the value of implied volatility as *Z*-axis and the time to maturity, *T*, as X-axis while the value of moneyness is set as Y-axis. From Figure 4.5 we can state that as the degree of moneyness becomes greater the implied volatility will decrease, which means implied volatilities for deep in-the-money calls and deep out-of-the-money puts are much higher than for implied volatilities for deep out-of-the-money calls and deep in-the-money puts. This illustrates the implied volatility smile is shaped like volatility skew of AAPL stock options. Moreover, as time to maturity increases, for options with the same exercise price, its implied volatilities are also increasing, especially for options tend to be at the money, which is consistent with previous statements.

Figure 4.5 Implied Volatility Surface for Bid



Source: Own calculation.

However, from Figure 4.5 we can find that the implied volatilities for options near expiration date tend to behave more volatile than those with longer time to maturities. Owing to many strict assumptions in the Black-Scholes pricing model, the theoretical volatility smile and the term structure of implied volatility display smooth and regular, but in real world, the prices of options are more sensitive than the theoretical prices because of many complex factors, such as the trading volume and the open interest. And this is also the reason for the existence of some extreme values.

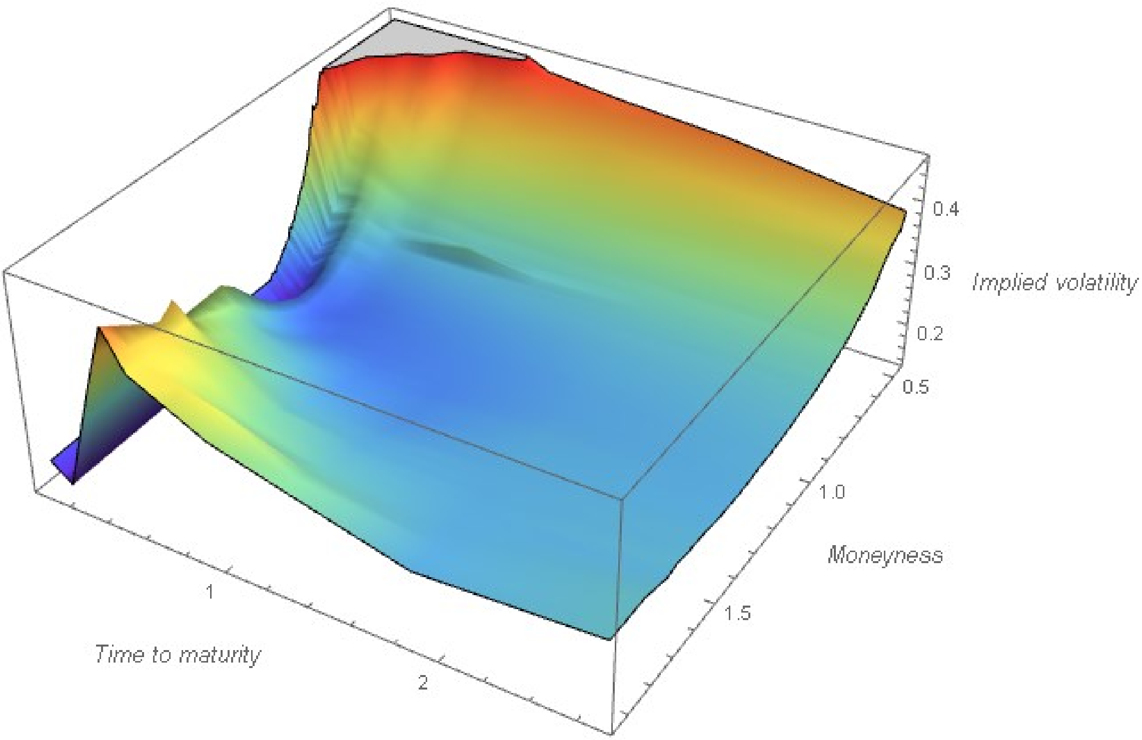
As the options approach maturity dates, there will be more and more traders deal with the options, especially for at-the-money options, therefore, the trading volumes will increase and at the same time the open interest, which refers to the number of outstanding contracts that have not been settled, will decrease. This shows the volatility which is implied by option itself is decreasing because of its maturation, thus relatively low implied volatilities for those at-the-money options near maturity date are displayed in the implied volatility surface.

The construction for implied volatility surface which is using implied volatilities calculated by ask prices is similar with the way that we applied before, and the result is shown in Figure 4.6. We can state from Figure 4.6 that its volatility smile behaves in the similar way compared with the volatility smile for bid prices. In other words, it means its implied volatility is decreasing as the degree of moneyness becomes greater. But the term structure of its implied volatility shows implied volatility is decreasing as time to maturity increases especially for those options tend to be at the money, which behaves much different from what we stated before. In order to figure out the reasons for this phenomenon, we need to understand the relationship between bid and ask prices of options.

The bid price represents the highest price that a buyer is willing to pay for a security or a contract, similarly, the ask price refers to the minimum price that a seller is willing to receive for a security or a contract. The difference between bid and ask prices is known as bid-ask spread, this indicator is a reflection of the liquidity of trading product.

Generally speaking, the lower the bid-ask spread, the better the liquidity of a security or contract; this is because the transaction only occurs when a buy and a seller agrees on a same price for the trading product. Therefore, the difference between implied volatilities that are calculated respectively using bid and ask prices can be caused by the different bid-ask spread for option prices with different time to maturity and exercise price. Thus, the market prices for options contain much more complex factors and information than the theoretical prices, which are calculated under many strict assumptions following the Black-Scholes model. As a result, the behavior of implied volatility, which is calculated using the market price, is more volatile and abnormal compared with the theoretical model.

Figure 4.6 Implied Volatility Surface for Ask



Source: Own calculation.

4.3. Pricing AAPL Stock Option

The first kind of options that we will price in the thesis is AAPL stock options with various expiration dates and exercise prices. In order to figure out what influence will be brought about to the option prices by implied volatilities, we apply three different kind of volatilities to price the options, which respectively are the historical volatility, at-the-money implied volatility and corresponding implied volatilities according to various exercise prices and expiration dates.

The date for pricing AAPL stock options is on March 12, 2016, and we choose six different expiration dates to price stock options, and the details can be found in Table 4.5. The data for historical volatility is collected from the website of CBOE and is considered as the mid value for bid price and ask price. In order to derive the historical volatility respectively for calculating bid prices and ask prices of options, we apply the volatility spread, which is calculated from the difference between the implied volatility for bid and ask price. Moreover, for at-the-money volatility, we adopt implied volatility for those at-the-money options with different expiration dates. And implied volatility is adopted by the interpolation from the implied surface constructed in the previous chapter.

Table 4.5 Pricing Factors

Pricing date	2016/3/12					
Current stock price	102.26 USD					
Expiration date	2016/4/15	2016/5/20	2016/6/17	2016/7/15	2016/8/19	2016/9/16
Time to Maturity	33	68	95	123	157	184
T (p.a.)	0.131	0.270	0.377	0.488	0.623	0.730
Riskless rate	0.091%	0.14%	0.22%	0.32%	0.36%	0.40%
Dividend yield	1.08%					
Historical volatility(mid)	24.87%	32.67%	31.32%	29.51%	29.18%	27.96%
Volatility spread	1.72%	1.71%	2.34%	2.16%	1.75%	1.60%
Historical volatility(BID)	24.01%	31.81%	30.15%	28.43%	28.31%	27.16%
Historical volatility(ASK)	25.73%	33.53%	32.49%	30.59%	30.05%	28.76%
ATM volatility(BID)	19.17%	20.44%	21.60%	21.28%	21.49%	21.79%
ATM volatility(ASK)	19.78%	20.92%	22.06%	21.72%	21.93%	22.22%

Source: CBOE and own calculation.

Using the data displayed in Table 4.5, we can price the AAPL stock options with various maturity dates and exercise prices. Here we take the prices for options with maturity date on April 15, 2016 as example and apply corresponding implied volatilities to price these options. The results are shown in Table 4.6, where the unit of prices is USD. It should be noted that the option price can be accepted only if it satisfies the non-arbitrage principle according to Equation (2.14) and (2.16). For instance, for options with exercise price of 98.2 USD, the call option price should be fall in range of $[4.35, 102.26]$ and the range for put option price should be $[0, 97.64]$; and its price for call and put respectively falls in its price range, thus it satisfies non-arbitrage pricing principle, our calculated prices can be considered as reasonable prices.

Furthermore, as shown in Table 4.6, when the exercise price increases, price for call option is decreasing while price for put option is increasing. This is because call options tend to become out of the money when exercise prices become higher, and at the same time put options tend to become in the money. And the ask prices are always higher than the bid prices, for options almost at the money, the bid-ask spread is small, and this indicates the high liquidity of ATM options, which is consistent with general conditions.

Thanks to the consideration of accuracy, we take prices for call options, which are almost at the money, as objects to analyze. For example, the bid prices for stock call options with maturity date on April 15, 2016, and its moneyness various from 0.795 to 1.35, which eliminates prices of deep in-the-money calls and deep out-of-the-money calls, is shown in Figure 4.7. We can state from Figure 4.7 that the option bid price curves calculated using these three different volatilities are almost coincident, except for some calls that are almost at the money, which is priced using historical volatility. This indicates that a stronger fluctuation is reflected by historical volatility and as a result, reflected as higher option prices.

Table 4.6 AAPL Stock Option Pricing

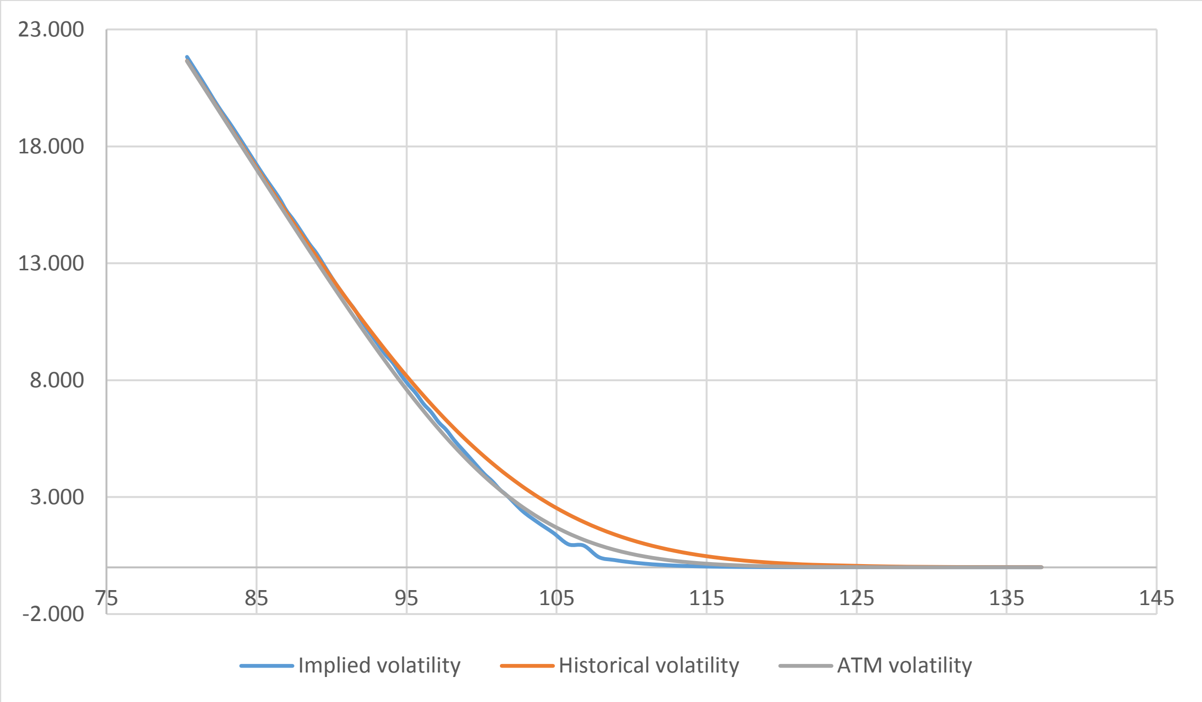
Expiration date: 2016/4/15						
BID				**ASK**		
σ_{im}	**CALL**	**PUT**	**K**	**CALL**	**PUT**	σ_{im}
21.85%	5.443	1.598	98.2	5.565	1.721	22.80%
21.58%	5.073	1.736	98.7	5.188	1.852	22.46%
21.30%	4.710	1.882	99.2	4.809	1.982	22.03%
20.88%	4.338	2.018	99.7	4.455	2.137	21.73%
20.47%	3.979	2.168	100.2	4.165	2.356	21.79%
20.41%	3.681	2.379	100.7	3.798	2.497	21.22%
19.86%	3.327	2.533	101.2	3.427	2.634	20.55%
19.75%	3.048	2.762	101.7	3.150	2.866	20.45%
18.60%	2.398	3.129	102.7	2.483	3.216	19.19%
18.19%	1.915	3.664	103.8	2.013	3.763	18.87%
17.63%	1.475	4.241	104.8	1.581	4.348	18.40%
16.11%	0.981	4.764	105.8	1.088	4.872	16.98%

Source: Own calculation.

On the left side of the cross point of option prices calculated by ATM volatility and corresponding implied volatilities, the price curve calculated using implied volatility is above the price curve calculated using ATM volatility; while on the right side of the cross point, it behaves opposite. The cross point of these two price curves is the price for at-the-money option; and the existence of this phenomenon is because the existence of implied volatility skew: as the exercise price increases, the implied volatility is decreasing. In other words, implied volatilities for in-the-money calls are greater than ATM implied volatility, and implied volatilities for out-of-the-money calls are less than ATM implied volatility, and the reflection of this phenomenon in option prices is expressed in Figure 4.7.

Since we apply the interpolations selected from implied volatility surface as the implied volatilities to price options, sometimes there will exist some “abnormal” values, which make its price curve not smooth. This can be found in Figure 4.7 where exercise prices between 105 USD and 110 USD, but this cannot be considered as a mistake. It is considered as the inconsistent between realistic and theoretical model because of the absence of some strict assumptions in real markets.

Figure 4.7 AAPL Stock Call Option Bid Prices with Maturity Date at April 15, 2016



Source: Own calculation.

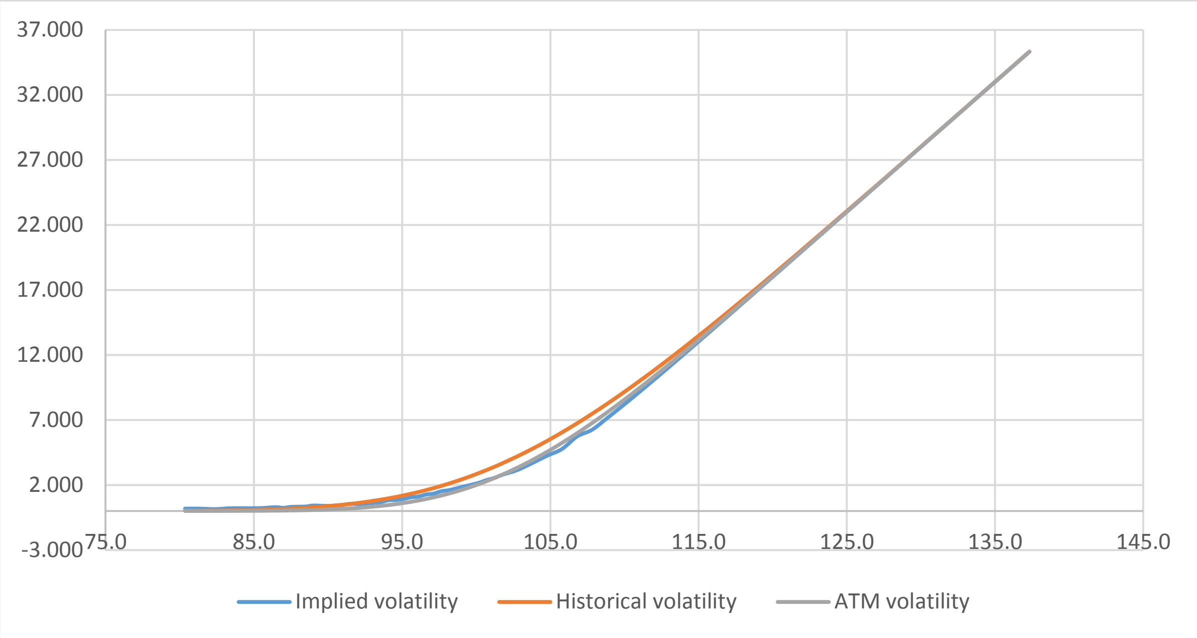
There exist a similar performance for AAPL stock put option bid prices with the same maturity date, which can be displayed in Figure 4.8. From Figure 4.8 we can state that the historical volatility also carries a higher fluctuation compared with other volatilities, thus the put options bid prices calculated using the historical volatility are greater than the remaining two. Therefore, if traders price options by using the historical volatility, there is possibility that they will overestimate option prices because of a higher volatility expectation.

However, unlike for call prices, for out-of-the-money puts, the bid prices which is calculated using implied volatility are greater than those derived from ATM volatility, and for in-the-money options, a completely opposite condition is held.

Although the implied volatility skew are almost the same for both calls and puts, the trading condition for calls and puts are exactly different: as exercise price decreases, the calls tend to become more and more in the money while the puts tend to become more and more out of the money and vice versa. And this is the reason why price curves for puts are upward sloping while price curves for calls are downward sloping.

For options with same exercise price but different expiration dates, there also exist some relationship among them. The put option price curves, which are calculated using three different volatilities, and with exercise price equals to 100.2 USD, are displayed in Figure 4.9.

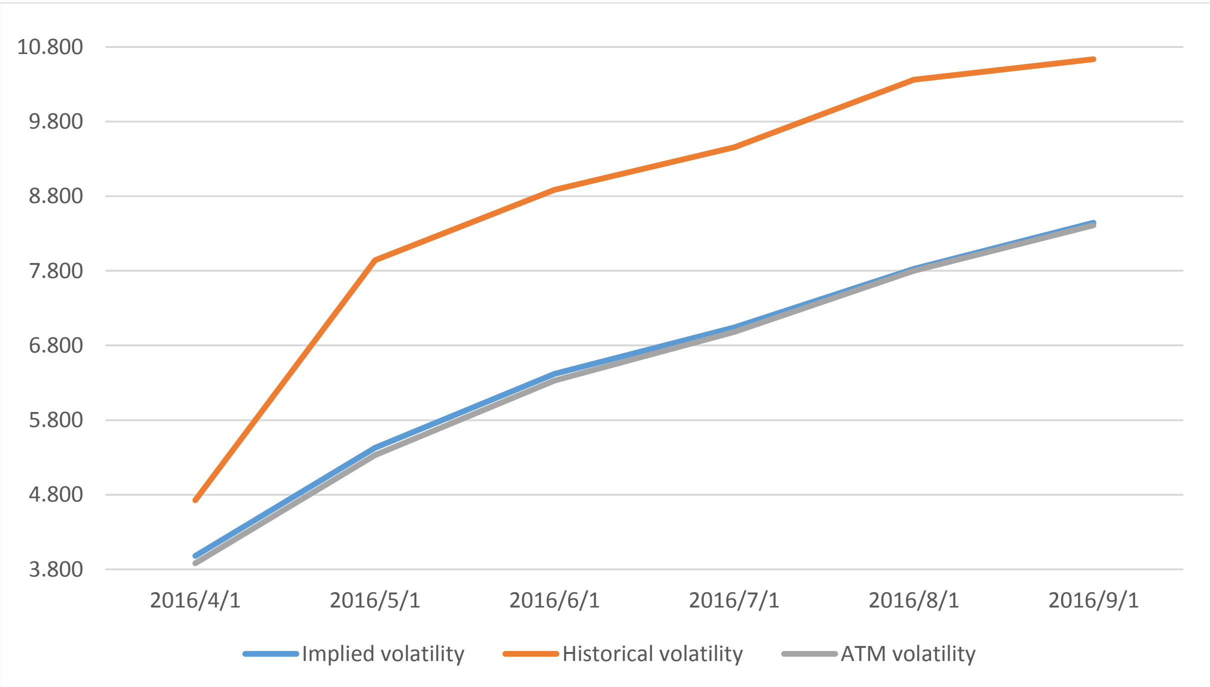
Figure 4.8 AAPL Stock Put Option Bid Prices with Maturity Date on April 15, 2016



Source: Own calculation.

From Figure 4.9, we can state that the option price curves calculated using ATM volatility and implied volatility are almost coincident because at this exercise price the options are almost at the money. In pace with the increasing of time to maturity, the option prices are also increasing due to the increase of options' time value.

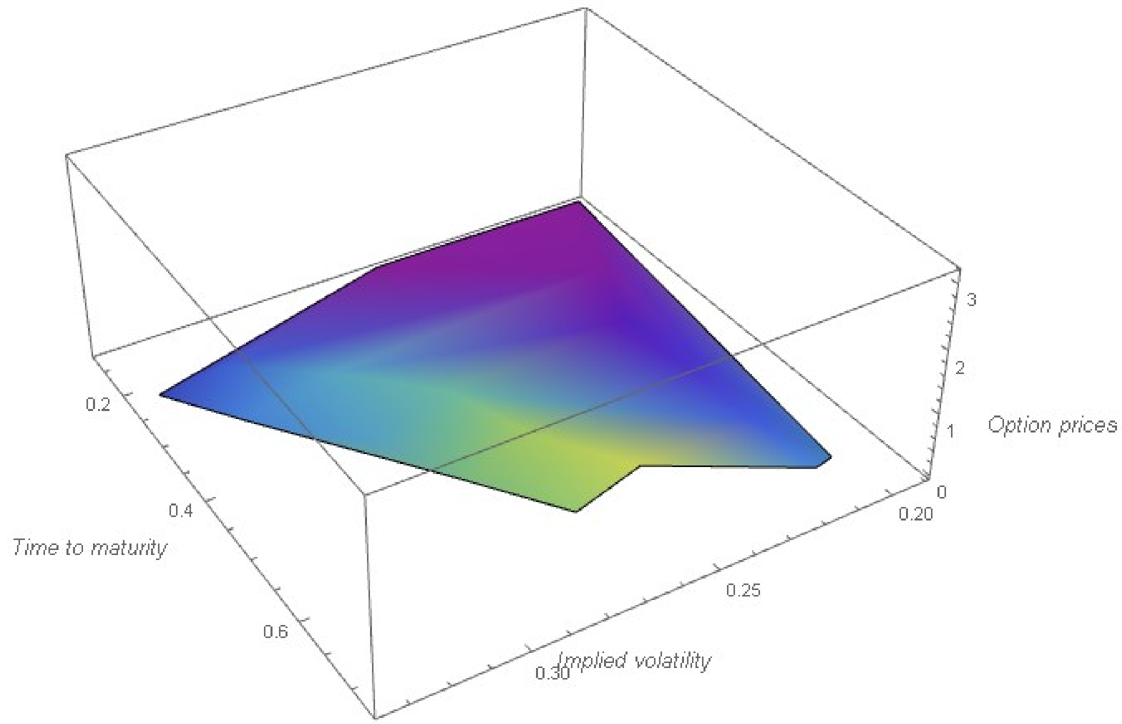
Figure 4.9 AAPL Stock Put Option Prices Term Structure



Source: Own calculation.

Compared with the remaining two price curves, the price curve calculated using historical volatility indicates relatively higher prices. This is because the historical volatility is kind of volatility which only takes the fluctuation of underlying stock price into consideration. The behavior of stock price is different with option price since it does not exist situation such as out of the money, at the money or in the money. The historical volatility only measures the extent that stock price fluctuates, it is unrelated to the exercise price level, but the value of implied volatility is related with the relationship between exercise price and current stock price. Furthermore, the historical volatility is constant for options with the same expiration date but different exercise prices, and this is its biggest disadvantage. Therefore, option price calculated using historical volatility may not as accurate and reliable as price that is determined by implied volatility.

Figure 4.10 Option Price Surface for AAPL Stock Put Option



Source: Own calculation.

For the purpose of connecting the relationship among volatility, time to maturity and option price together, we can construct an option price surface, which can indicate how the option price changes as time to maturity changes as well as the value of volatility. The way to construct option price surface is similar with the way we constructed implied volatility surface before.

For instance, the option price surface for AAPL stock put option is displayed in Figure 4.10.

In option price surface, we set time to maturity, T, as X-axis while the value of implied volatility as Y-axis, and option price that calculated following Black-Scholes pricing formula as Z-axis.

From Figure 4.10 we can state that for AAPL stock put options, as the time to maturity increases, its option price will increase as well, and this illustrates the conclusion that we observed in Figure 4.9. Moreover, the relationship between implied volatility and option prices can be observed in the option price surface as well: if hold time to maturity constant, then option price will increase when the volatility increases and vice versa. By constructing option price surface, we can figure out the relationship between the option prices and its influence factors more clearly and conveniently.

To sum up, if we apply the historical volatility to price the stock options, the results will be less reliable since the historical volatility only measures the fluctuation in the underlying stock prices, there are other factors which can influence prices of stock options, for example the trading volume. Generally speaking, option prices that calculated using the historical volatility are probably overestimated. Moreover, the historical volatility is constant for options with different exercise prices but the same expiration date, this is illogical for pricing options as well.

The most accurate way to price stock options is to use implied volatility, which is calculated according to the Black-Scholes-Merton model. For options with different exercise prices and time to maturity, we should apply different implied volatility which is calculated in the similar circumstance. The prices of stock options that calculated using ATM volatility is similar to the prices calculated using implied volatility, and it is more reliable compared with the prices calculated using the historical volatility, but its biggest disadvantage is similar with the historical volatility, which means it holds the constant volatility for options with different exercise prices but the same expiration date. As a result, it will probably overestimate the value of out-of-the-money calls and in-the-money puts, and underestimate the value of in-the-money calls and out-of-the-money puts because of the existence of volatility skew.

4.4. Pricing Floating Lookback Option

For pricing floating lookback option, we similarly apply two different volatilities to price, which respectively are historical volatility and ATM implied volatility. Since it does not exist specific exercise price, we will not price floating lookbacks by using corresponding implied volatilities. The pricing process of floating lookback option is followed by Equation (2.40) and (2.44), which is introduced in the previous chapter.

For simplicity, we assume the floating lookback option is just originated since the pricing day, therefore we can apply the assumption that $S_{min} = S_{max} = S_0$. And the pricing date is on March 12, 2016.

Table 4.7 Floating Lookback Option Price Calculated by Historical Volatility

$\sigma_{historical}$	BID		Expiration date	ASK		$\sigma_{historical}$
	Call	Put		Call	Put	
24.01%	3.415	7.402	2016/4/15	3.667	7.939	25.73%
31.81%	6.608	14.293	2016/5/20	6.947	15.117	33.53%
30.15%	7.411	16.087	2016/6/17	7.909	17.470	32.49%
28.43%	7.939	17.324	2016/7/15	8.487	18.762	30.59%
28.31%	8.892	19.638	2016/8/19	9.406	20.958	30.05%
27.16%	9.198	20.458	2016/9/16	9.691	21.789	28.76%

Source: Own calculation.

Other factors that are needed for calculating prices can be found in Table 4.5. And the floating lookback option prices calculated using historical volatility, is shown in Table 4.7, where the unit of prices is USD. We can state from Table 4.7 that as time to maturity increases, the prices for both floating calls and puts are also increasing due to the increases of the time values of options.

Table 4.8 Floating Lookback Option Price Calculated by ATM Volatility

σ_{ATM}	BID		Expiration date	ASK		σ_{ATM}
	Call	Put		Call	Put	
19.17%	2.702	5.905	2016/4/15	2.791	6.093	19.78%
20.44%	4.207	9.060	2016/5/20	4.287	9.297	20.92%
21.60%	5.280	11.372	2016/6/17	5.314	11.695	22.06%
21.28%	5.916	12.804	2016/7/15	5.980	13.127	21.72%
21.49%	6.715	14.702	2016/8/19	6.816	15.052	21.93%
21.79%	7.345	16.223	2016/9/16	7.436	16.606	22.22%

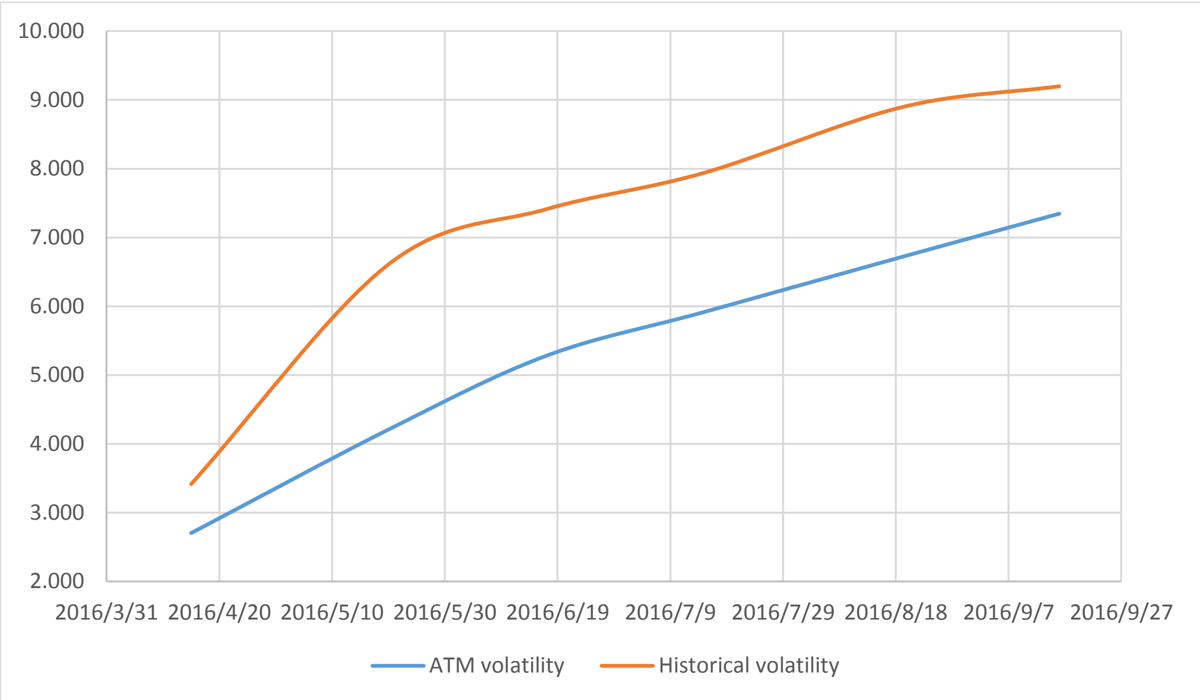
Source: Own calculation.

The floating lookback option is different from plain vanilla options, and it does not have specific exercise price. The payoffs of floating lookback options are depending on the difference between the maximum or minimum price that its underlying stock reached during the life of option and the stock price on the maturity date. In other words, the situation for in-the-money and out-of-the-money option is not typical for floating lookback option. Therefore, historical volatility that measures the fluctuation of stock prices can be considered as a reasonable volatility for pricing lookbacks.

Due to the absence of exercise price, we cannot apply corresponding implied volatility to price floating lookbacks. For simplicity, we have set the assumption that $S_{min} = S_{max} = S_0$, therefore, the most appropriate implied volatility for pricing floating lookback options should be ATM implied volatility, whose value is calculated backward from at-the-money option with corresponding expiration date. And the result for floating lookback prices which are determined by ATM implied volatility is shown in Table 4.8, where the unit for prices is USD.

From Table 4.8, we can state that the prices of floating lookback options behave almost in the same way as the prices that calculated using historical volatility, which shows the increasing tendency for both calls and puts as the time to maturity increases. In order to figure out the difference between these two pricing method, its price curves are displayed in Figure 4.11 to demonstrate.

Figure 4.11 Floating Lookback Calls Bid Prices Curves



Source: Own calculation.

We can state from Figure 4.11 that the floating lookback call prices calculated using historical volatility are relatively greater than the prices determined by ATM volatility. As we stated before, ATM implied volatility measures the volatility for at-the-money options, but in general, we cannot consider floating lookback options as at-the-money options. Because stock price on the maturity date is not clear nor do the maximum or minimum stock prices during the life of option.

But in our case, we consider the assumption that $S_{min} = S_{max} = S_0$ for simplicity, which means we can consider the floating lookback option as ATM option under this assumption. Therefore, in our case, the prices of floating lookbacks calculated using ATM volatility are more reliable than the prices calculated using the historical volatility.

In conclusion, unlike stock options, floating lookback option does not have specific exercise price, and its value is determined primarily on the maximum or minimum prices of underlying stock observed during the life of the option. Therefore, the historical volatility, which reflects the fluctuation of stock prices, can be considered as a reasonable volatility to price floating lookback option. Furthermore, under the assumption that $S_{min} = S_{max} = S_0$, ATM volatility can be considered as a more reliable volatility compared with the historical volatility.

4.5. Pricing Fixed Lookback Options

Similarly with the pricing process for floating lookback options, we adopt the assumption that the fixed lookback option is just originated since the pricing day as well, therefore we can consider the assumption that $S_{min} = S_{max} = S_0$ for simplicity. And for fixed lookback options, we adopt three different kind of volatilities to price, which respectively are the historical volatility, at-the-money volatility and corresponding implied volatilities according to various exercise prices and expiration dates.

Table 4.9 Fixed Lookback Option Prices Calculated by Implied Volatility

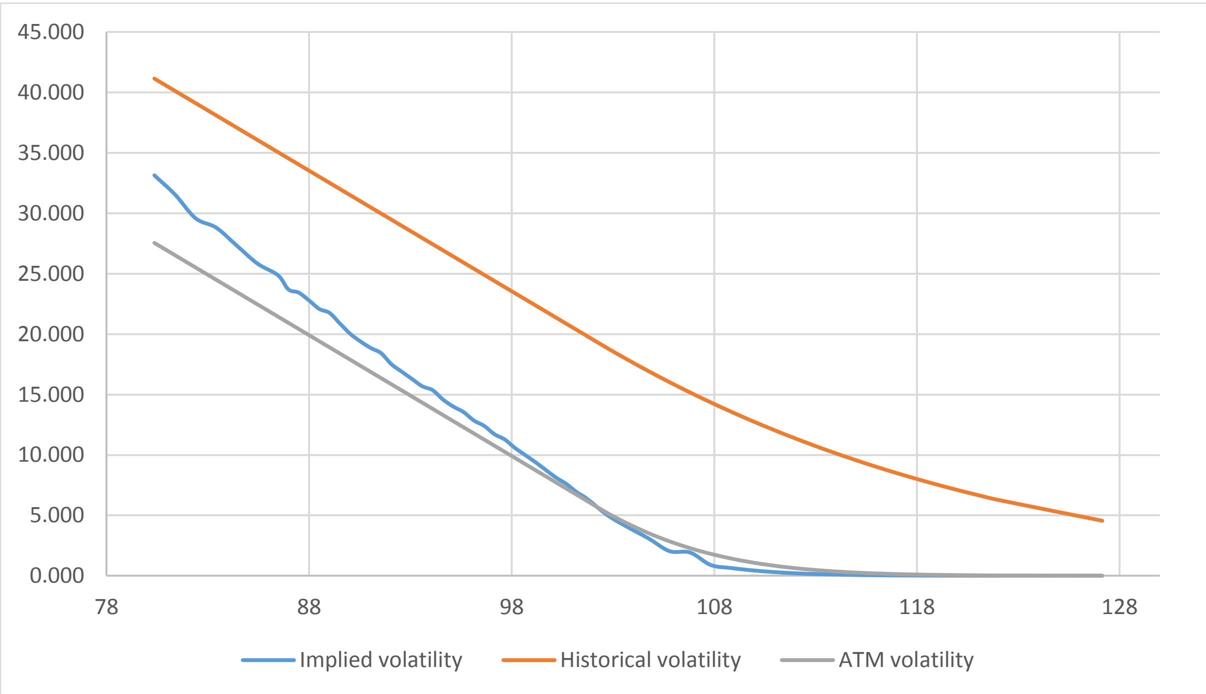
Expiration date: 2016/4/15						
BID				**ASK**		
σ_{im}	**CALL**	**PUT**	**K**	**CALL**	**PUT**	**σ_{im}**
22.25%	11.719	3.862	97	12.013	4.122	23.20%
22.48%	11.281	4.180	98	11.577	4.444	23.44%
21.85%	10.576	4.263	98	10.870	4.528	22.80%
21.58%	9.986	4.452	99	10.256	4.695	22.46%
21.30%	9.390	4.637	99	9.616	4.842	22.03%
20.88%	8.750	4.783	100	9.012	5.022	21.73%
20.47%	8.117	4.936	100	8.523	5.306	21.79%
20.41%	7.588	4.451	101	7.839	4.681	21.22%
19.86%	6.912	4.807	101	7.124	5.002	20.55%
19.75%	6.367	5.282	102	6.584	5.481	20.45%
18.60%	4.998	5.976	103	5.177	6.142	19.19%

Source: Own calculation.

The pricing process for fixed lookback option, which is calculated using implied volatility, is similar as for AAPL stock option. It means we also take the interpolations from the implied volatility surface as the corresponding implied volatilities to price the fixed lookback options. And the results of fixed lookback options' prices that are determined by implied volatilities are shown in Table 4.9, where the unit for prices is USD.

From Table 4.9 we can state that the prices for fixed lookbacks that tend to be at the money are much higher compared with the prices for AAPL stock options that tend to be at the money. This is mainly because the lookback options can bring more profit to traders compared with plain vanilla options. The traders can profit from fixed lookback calls when S_{max} exceeds exercise price while profit from lookback puts when exercise price exceeds S_{min}. And their profits are the difference between the extreme stock price and the exercise price. But for plain vanilla options, the profits for traders are the difference between stock price at the maturity date and its exercise price. Obviously, traders that hold fixed lookbacks are more likely to earn greater profit than traders that hold plain vanilla options; therefore, even for at-the-money options, prices of lookbacks are relatively higher than prices of stock options.

Figure 4.12 Fixed Lookback Call Bid Price Curves with Marturity Date on April 15, 2016

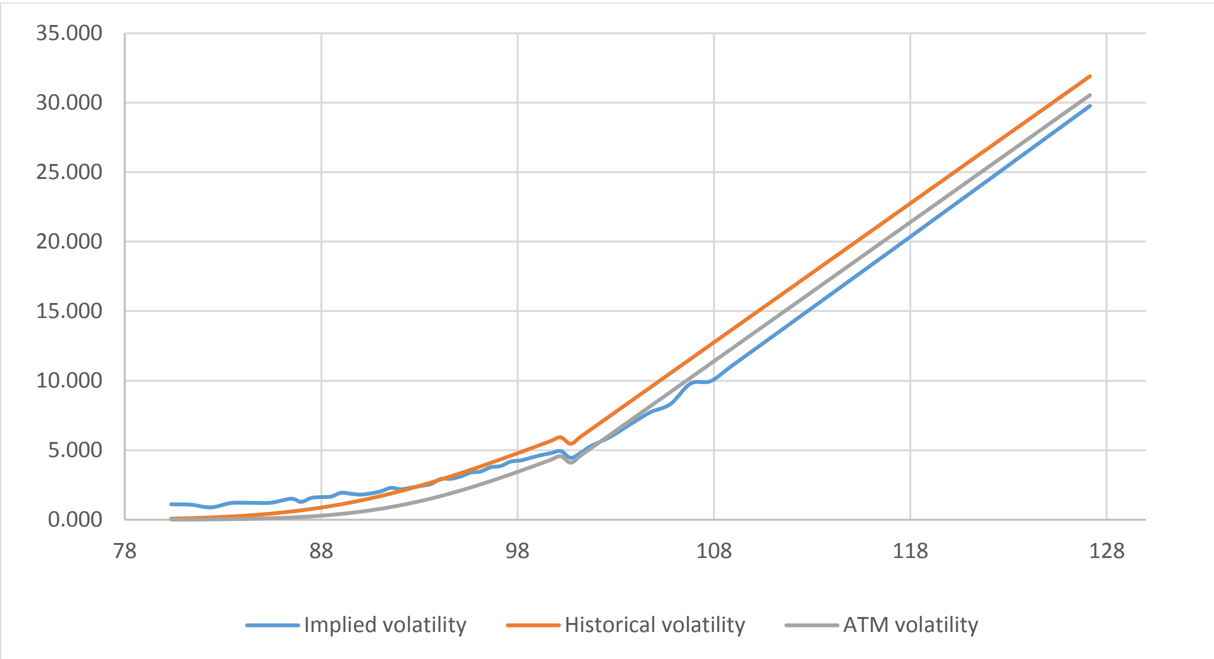


Source: Own calculation.

The comparison among the lookback call price curves calculated using different volatilities, and with maturity date on April 15, 2016, are displayed in Figure 4.12. We can conclude from Figure 4.12 that the price curves calculated using ATM volatility and implied volatility are almost coincident, while the price curve calculated using historical volatility lies above the remaining two price curves. The price curve calculated using implied volatility for in-the-money lookback calls lies above the price curve calculated using ATM volatility, and it lies below price curve calculated using ATM volatility for out-of-the-money lookback call simultaneously; and this is due to the existence of implied volatility skew.

However, the condition turns to be a little different for fixed lookback puts. The bid price curves of fixed lookback puts with maturity date on April 15, 2016 are displayed in Figure 4.13. We can state from Figure 4.13 that the price curve calculated using historical volatility is not laying above the remaining two price curves all the time. It intersects with price curve calculated using imlied volatility before reaching at-the-money exercise price. This is mainly because the historical volatility is constant for various exercise prices while the implied volatility changes for specified exercise prices. Hence, we can conclude that the implied volatility has a higher expectation for volatility of out-of-the-money lookback puts compared with its historical volatility.

Figure 4.13 Fixed Lookback Put Bid Price Curves with Maturity Date on April 15, 2016



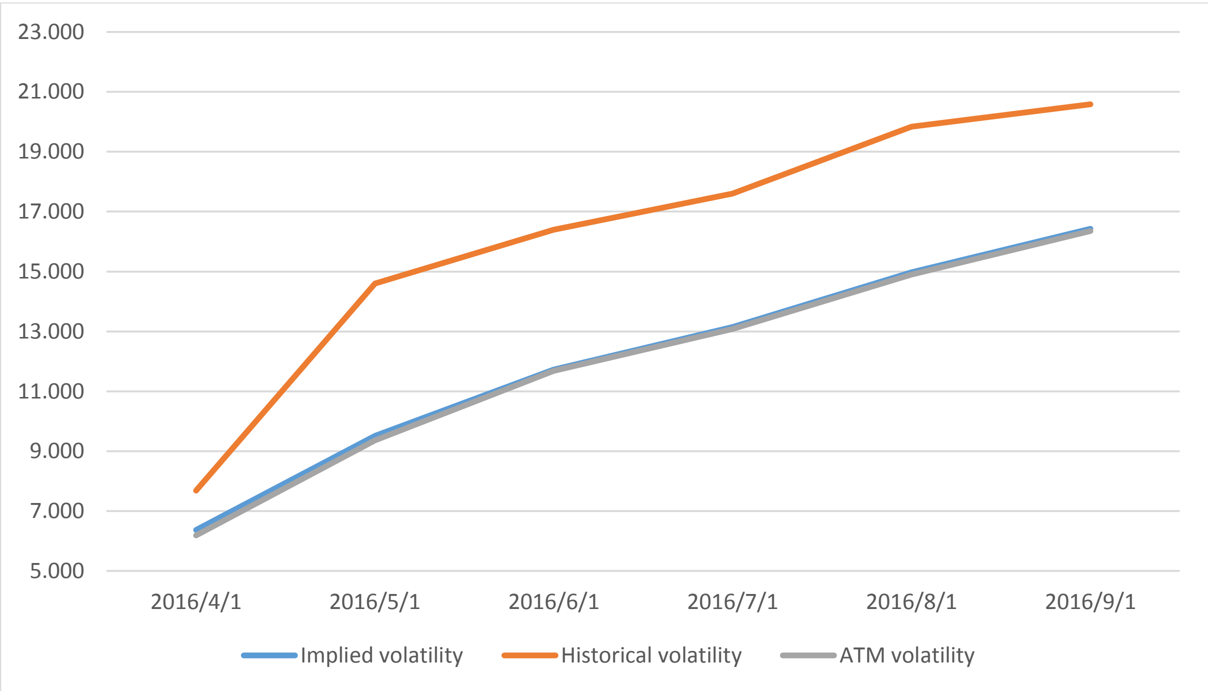
Source: Own calculation.

Since lookback option is a kind of exotic option, its pricing process is more complex than plain vanilla options, and if we set the rules more strictly: the implied volatility surface only determine an appropriate implied volatility to substitute into BS pricing model when pricing plain vanilla options. In other words, the implied volatility that be found in the volatility surface is not appropriate to be considered as a liable volatility to price exotic options. However, for the purpose of simplicity, we simply apply the implied volatility calculated according to BS model to price lookback options. Therefore, the lookback option prices that determined by corresponding implied volatility are not liable enough and its price curve behaves flucuated compared with the remaining two price curves.

On account of the existence of implied volatility skew, the price curve calculated using implied volatility lies above the price curve calculated using ATM volatility before exercise price reaches the current stock price, and lies below price curve calculated using ATM volatility on the righet side of ATM exercise price.

From both Figure 4.12 and Figure 4.13 we can figure out that the price variation ranges of fixed lookback options are greater than plain vanilla options, this can be caused by the difference between their profitabilities. As we stated before, under the condition of same exercise price and expiration date, there is possibility for fixed lookback options to generate greater payoffs compared with stock options; therefore, fixed lookback option is considered to be more valuable than stock option when it is in the money.

Figure 4.14 Term Structure of Fixed Lookback Call Bid Prices



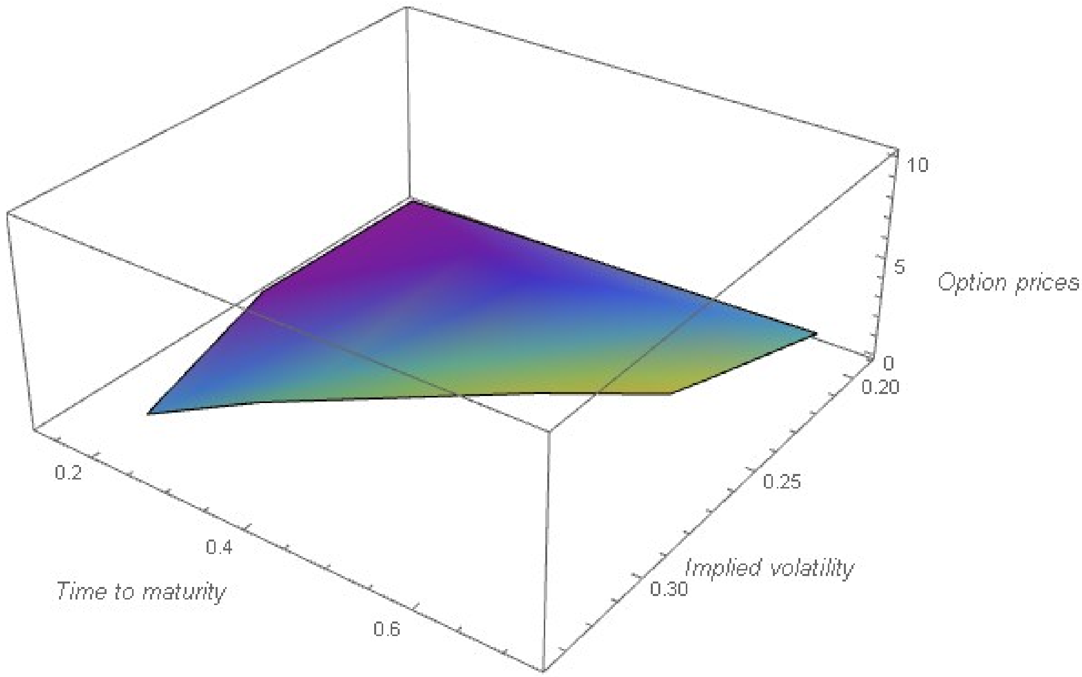
Source: Own calculation.

Nevertheless, when it turns to out-of-the-money options, the situation will be totally opposite. Since out-of-the-money option will not be exercised on maturity date, the price for out-of-the-money option represents the loss of traders. For example, as for fixed lookback puts, if the lowest price of underlying stock is 90 USD, then there is small possibility that during life of option the underlying stock price will fall below 90 USD; hence, for fixed lookback puts with exercise prices less than 90 USD, the possibility that it can be exercised is extemely small, correspondingly, the value of these puts will be worthless.

Afterwards, we can examine the relationship between fixed lookback option prices and time to maturity. The term structure of fixed lookback calls bid price on various maturity dates is displayed in Figure 4.14. For the consideration of accuracy, we construct the price curves with prices of options that tend to be at the money.

We can state from Figure 4.14 that using historical volatility to price ATM options may relatively overestimate its value. Moreover, for all pricing method the situation holds the same: as the time to maturity increases, the prices of fixed lookback options are also increasing on account of the existence of its time value.

Figure 4.15 Option Price Surface for Fixed Lookback Put Options



Source: Own calculation.

Similarly, we can construct the option price surface for fixed lookback optioins as well. The opion price surface for fixed lookback puts that tend to be at the money is displayed in Figure 4.15. We can state from Figure 4.15 that it is similar with plain vanilla option price evolution, for fixed lookback option, as time to maturity increases, its price will increase as well due to the increase of its time value. Furthermore, if the value of time to maturity is held constant, then the option price will increase when implied volatility increases and vice versa.

Compared with stock option price surface, which is displayed in Figure 4.10, the most difference in fixed lookback option price surface is the range of its Z-axis. As we explained before, since the lookbacks generate more than plain vanilla options, which means it has a higher profitability, it is more valuable than stock option with same strike price and time to maturity as well. The price for fixed lookback puts that tend to be at the money is displayed in Figure 4.15, and we can state from Figure 4.15 that it illustrates a higher price compared with stock option price. However, if compared with out-of-the-money option, then lookback puts will tend to be worthless than the stock put options.

To sum up, for fixed lookback options, the prices calculated using historical volatility will probably overestimate the value of fixed lookbacks. In addition, the prices calculated using ATM volatility may overestimate the value of out-of-the-money calls and in-the-money puts, and it will also underestimate the value of in-the-money calls and out-of-the-money puts. These two volatilities cannot be considered as the reliable volatility to price lookback options because it holds constant for options with different exercise prices. The most accurate and reliable way to calculate the value of fixed lookback options is not clear because of the complex pricing process of exotic options. Nevertheless, if we loosen the rules for pricing exotic options, implied volatility can still be the most reliable pricing method compared with the remaining two volatilities because it adjusts specific volatilities for options with different exercise prices and time to maturities. Moreover, compared with stock options, fixed lookback options tend to have higher prices because of higher profitability.

4.6. Summary

Chapter 4 is the application part of the thesis, in this chapter, we collected market data and then priced AAPL stock option, floating lookback option and fixed lookback option. Moreover, by calculating implied volatilities that can be fed into BS pricing model, we constructed the implied volatility surface, which is useful for us when pricing options.

By analyzing the existed market prices of AAPL stock options with various expiration dates and exercise prices, we can conclude that as strike price increases, the market prices for calls will decrease while the market prices for puts will increase. And for both call and put options with the same strike price, when the value of time to maturity increases, option price will also increase.

After obtaining the market prices of AAPL stock options, we calculated its implied volatility and constructed the implied volatility surface respectively for bid price and ask price. The implied volatility surface is the connection of volatility smile and the term structure of implied volatility. The volatility smile derived from real market data behaves like a volatility skew, which means a higher strike price will lead to a relatively lower implied volatility and vice versa. The term structure of implied volatility is shown as a rising curve, which illustrates that the short term implied volatility is expected to rise and the prices of short term options are also expected to rise.

We priced AAPL stock options with expiration dates range from April 15 to September 16, 2016 respectively using implied volatility surface, ATM volatility and historical volatility. The results reflected that using historical volatility to price stock option will overestimate the value of option compared with the remaining volatilities. And if we use implied volatility to price stock options, there may exist some abnormal prices that will make its price curve not smooth, this is because of the complicated real market environment. If we apply the historical volatility or ATM volatility to price the stock options, the results will be less reliable because these two volatilities are constant for different exercise prices. But we can apply different implied volatility which is calculated in the similar circumstance for options with different exercise prices and time to maturity. Hence, the most accurate way to price stock option is to use the corresponding implied volatility.

Then we priced both floating and fixed lookback options, and assumed the lookback option is just originated since the pricing day for simplicity during the pricing process. For floating lookbacks which do not have exercise price, we considered both of the historical volatility and ATM volatility as the reliable volatility to price floating lookbacks. Nevertheless, if the assumption that $S_{min} = S_{max} = S_0$ is applied, then the price calculated using ATM volatility is more accurate compared with the price calculated using the historical volatility.

And for fixed lookbacks, its price curves behave in the similar way compared with stock option price curve, except for a broader range of option prices. This is because exotic option tend to have a higher profitability than plain vanilla option. We also constructed price surface for both stock options and fixed lookbacks to illustrate the relationship among option prices, volatilities and time to maturities clearly.

In conclusion, the pricing process of exotic option is more complicated than plain vanilla option, thus implied volatility calculated backward from BS model cannot be considered as a kind of accurate volatility to price fixed lookback option. However, under the simplified assumption, implied volatility has taken more factors which can influence the option prices into consideration, and it adjusts the volatilities for different exercise prices and time to maturities. Therefore, it can still be applied to price fixed lookback option, and its results are considered to be more accurate than prices calculated using historical volatility and ATM volatility, which are constant for different exercise prices.

5. Conclusion

Derivatives are playing a significant role in financial markets throughout the world, and option, as one of the most important derivatives, are becoming increasingly popular and attractive towards traders nowadays. In order to take advantage of options, option pricing is a problem that concerned by mostly traders.

In order to reach the goal of the thesis, which is to examine the existence of implied volatility smile and construct implied volatility surface by using the real market data, we firstly studied the basic conception of option and its pricing principles and pricing model in Chapter 2. Through introducing the fundamental characteristics of options and some principles of option pricing, which includes put-call parity, risk-neutral pricing theory and non-arbitrage pricing principle, we learned theoretical basis that help us with option pricing. Moreover, the introduction of Black-Scholes-Merton model are highlighted in Chapter 2 as a fundamental pricing model for stock options. The stochastic process for stock price, Itô's lemma and the Black-Scholes-Merton differential equation, which are the fundamentals of the Black-Scholes-Merton model, are also presented in this chapter. In the end of Chapter 2, we derived Black-Scholes pricing formula and then extended the Black-Scholes pricing formula to price options on stock paying dividends. And the pricing formula for both floating and fixed lookback option are presented respectively as well.

In Chapter 3, we introduced the idea of implied volatility. By comparing implied volatility with the historical volatility and introducing the calculation of implied volatility and VIX, we generated a better understanding of this theory. The most significant essential in the thesis is the idea of implied volatility smile, which are highlighted in Chapter 3. We explained why volatility smile is the same for both call and put options and cause for the existence of volatility smile in this part. Last but not the least, we presented the term structure of implied volatility and the construction of implied volatility surface.

According to the analysis in Chapter 4, we found the option prices that calculated using implied volatility behave almost coincident with the market prices. The existed market prices of AAPL stock options show the pattern that as strike price increases, the market prices for calls will decrease while the market prices for puts will increase. And for both calls and puts with the same strike price, when the value of time to maturity increases, option price will also increase. And the prices that we calculated for both stock options and lookback options show the similar condition compared with the market prices of AAPL stock options.

We priced AAPL stock options with expiration dates range from April 15 to September 16, 2016 respectively using implied volatility surface, ATM volatility and historical volatility. The results reflected that using historical volatility to price stock option would overestimate the value of option compared with the remaining volatilities. The prices calculated using ATM volatility will probably overestimate the value of out-of-the-money calls and in-the-money puts, and underestimate the value of in-the-money calls and out-of-the-money puts because of the existence of volatility skew. Since these two volatilities are constant for options with different exercise prices but same expiration date, we cannot consider these volatilities as reliable volatilities. The most accurate way to price stock options is to use implied volatility, because for options with different exercise prices and time to maturity, we can apply different implied volatility which is calculated in the similar circumstance.

Under the assumption that the lookback option is just originated since the pricing day for simplicity during the pricing process, we priced both floating and fixed lookbacks. For floating lookbacks which do not have exercise price, we considered both of the historical volatility and ATM volatility are reliable volatility to price. Nevertheless, if the assumption that $S_{min} = S_{max} = S_0$ is applied, then floating lookbacks can be considered as ATM options, therefore, price calculated using ATM volatility is more accurate compared with the price calculated using the historical volatility.

In addition, for fixed lookbacks, its price curves behave in the similar way compared with stock option price curve, except for a broader range of option prices, which is caused by the higher profitability of exotic options. Similarly, for fixed lookbacks, under the assumption that we loosen the rules for pricing process, the most reliable way to price fixed lookback options is to use implied volatilities, which are accommodated according to different exercise prices and time to maturity.

Moreover, the volatility smile derived from real market data behaves like a volatility skew, which means a higher strike price will lead to a relatively lower implied volatility and vice versa. The term structure of implied volatility is shown as a rising curve, which illustrates that the short term implied volatility is expected to rise and the prices of short term options are also expected to rise.

One of the biggest shortcomings of the approach adopted in the thesis is the simplicity of assumptions when pricing exotic options. We can apply the implied volatility surface to overcome the weakness of Black-Scholes-Merton model when pricing plain vanilla options, but for exotic options we might consider a more complex model to price them. Nevertheless, we can consider implied volatility as the most suitable volatility to price both plain vanilla options and exotic options, since it has been adjusted for various time to maturities and exercise prices.

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List of Abbreviations

K: strike price or exercise price.

r: riskless rate, i.e. yield of U.S government bond.

T: time to maturity.

S_0: current stock price.

S_T: stock price at time t.

S_{min}: the minimum stock price observed during the life of the option.

S_{max}: the maximum stock price observed during the life of the option.

c: value of European call option.

p: value of European put option.

C: value of American call option.

P: value of American put option.

q: dividend yield of the underlying stock.

D: the present value of dividend payment.

b: cost of rate.

σ: volatility or the standard deviation.

σ_{im}: implied volatility.

$\sigma_{historical}$: historical volatility.

N(x): cumulative probability distribution function for a standardized normal distribution.

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Ostrava dated 20. 4. 2016

Minyi Dong 董珉怡

Student´s name and surname

List of Annexes

Annex 1: AAPL option market prices

expiration date: 18 March, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
1	49.9	50.25	**50**	0.01	0.01	1
1	44.9	45.2	**55**	0.01	0.01	2
5	39.9	40.15	**60**	0.01	0.01	3
2	34.95	35.25	**65**	0.01	0.03	18
12	29.95	30.25	**70**	0.01	0.04	10
120	24.8	25.1	**75**	0.01	0.05	12
0	24.45	24.75	**76**	0.01	0.02	202
0	23.4	23.6	**77**	0.01	0.02	10
0	22.45	22.7	**78**	0.01	0.03	15
13	21.45	21.65	**79**	0.02	0.03	2160
466	20.4	20.6	**80**	0.03	0.07	760
0	19.35	19.55	**81**	0.02	0.04	311
0	18.45	18.7	**82**	0.03	0.04	35
0	17.45	17.7	**83**	0.03	0.05	15
0	16.45	16.65	**84**	0.03	0.05	35
26	15.45	15.65	**85**	0.08	0.11	220
1	14.95	15.2	**85.5**	0.08	0.11	10
1	14.55	14.85	**86**	0.09	0.12	20
0	14.05	14.25	**86.5**	0.09	0.12	1
12	13.55	13.75	**87**	0.1	0.13	26
2	13.05	13.35	**87.5**	0.11	0.13	418
5	12.6	12.8	**88**	0.12	0.16	41
2	12	12.2	**88.5**	0.13	0.17	2
4	11.5	11.75	**89**	0.14	0.18	35
9	11.05	11.25	**89.5**	0.16	0.2	6
145	10.55	10.75	**90**	0.17	0.22	3841
72	10.1	10.3	**90.5**	0.2	0.24	5
4	9.6	9.8	**91**	0.22	0.26	71
10	9.15	9.35	**91.5**	0.24	0.29	14
2	8.7	8.85	**92**	0.27	0.32	310
36	8.25	8.4	**92.5**	0.31	0.35	157
8	7.85	8	**93**	0.34	0.39	73
1	7.4	7.55	**93.5**	0.39	0.44	93
2	6.95	7.1	**94**	0.45	0.49	69
8	6.4	6.6	**94.5**	0.51	0.54	21

534	6.1	6.25	**95**	0.56	0.61	1459
20	5.7	5.8	**95.5**	0.65	0.69	123
150	5.25	5.4	**96**	0.73	0.77	87
4	4.8	4.9	**96.5**	0.81	0.87	144
445	4.45	4.6	**97**	0.94	0.98	246
526	4.1	4.25	**97.5**	1.05	1.1	854
368	3.7	3.8	**98**	1.19	1.24	3081
127	3.3	3.45	**98.5**	1.35	1.61	231
419	3.05	3.15	**99**	1.51	1.56	1316
294	2.68	2.75	**99.5**	1.69	1.75	1205
9058	2.46	2.55	**100**	1.88	1.95	3288
			100.53			
2989	1.95	2	**101**	2.36	2.42	2574
21240	1.5	1.54	**102**	2.88	2.97	1386
10711	1.11	1.15	**103**	3.5	3.6	212
8381	0.78	0.81	**104**	4.2	4.3	71
4174	0.55	0.59	**105**	4.95	5.1	204
439	0.39	0.4	**106**	5.75	5.9	104
355	0.26	0.28	**107**	6.65	6.85	15
90	0.17	0.2	**108**	7.55	7.75	2
72	0.11	0.15	**109**	8.5	8.7	31
1639	0.07	0.1	**110**	9.45	9.65	187
2	0.04	0.05	**111**	10.4	10.65	8
5	0.01	0.05	**112**	11.3	11.6	1
20	0.01	0.04	**113**	12.35	12.65	10
2	0.01	0.04	**114**	13.3	13.65	1
210	0.01	0.05	**115**	14.3	14.65	6
1	0.01	0.03	**116**	15.4	15.65	32
1	0.01	0.03	**117**	16.4	16.65	1
242	0.01	0.03	**118**	17.3	17.6	1
1	0.01	0.03	**119**	18.3	18.65	2
10	0.01	0.03	**120**	19.4	19.65	30
4	0.01	0.03	**121**	20.3	20.6	4
5	0.01	0.03	**125**	24.4	24.6	5
17	0.01	0.02	**130**	29.3	29.6	1
106	0.01	0.01	**135**	34.3	34.6	1
3	0.01	0.01	**140**	39.05	39.65	2
1286	0.01	0.01	**145**	44.3	44.6	120
105	0.01	0.01	**150**	49.3	49.6	3

53	0.01	0.02	**155**	54.3	54.6	4
100	0.01	0.02	**160**	59.3	59.6	400
104	0.01	0.02	**165**	64.3	64.65	4
70	0.01	0.02	**170**	69.3	69.6	9
2	0.01	0.02	**175**	74.3	74.6	150

expiration date: 15 April, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
53	45.45	45.75	**55**	0.01	0.01	2
90	40.5	40.8	**60**	0.01	0.02	10
1	35.5	35.8	**65**	0.02	0.04	88
20	30.5	30.75	**70**	0.05	0.06	36
14	25.5	25.7	**75**	0.08	0.09	19
4	20.55	20.75	**80**	0.15	0.16	1130
0	16.6	16.85	**84**	0.16	0.19	0
10	15.65	15.85	**85**	0.26	0.27	721
0	15.1	15.35	**85.5**	0.13	0.17	0
0	14.6	14.85	**86**	0.21	0.25	0
0	14.15	14.35	**86.5**	0.23	0.27	0
0	13.65	13.95	**87**	0.25	0.29	0
8	13.3	13.5	**87.5**	0.37	0.38	94
0	12.75	13.1	**88**	0.28	0.33	0
1	12.3	12.6	**88.5**	0.31	0.36	0
0	11.85	12.05	**89**	0.35	0.38	0
0	11.4	11.65	**89.5**	0.38	0.42	0
96	11.05	11.2	**90**	0.53	0.57	1435
0	10.4	10.65	**90.5**	0.45	0.49	0
0	10	10.2	**91**	0.49	0.53	0
2	9.55	9.75	**91.5**	0.53	0.57	0
0	9.1	9.3	**92**	0.58	0.62	0
324	8.8	8.9	**92.5**	0.81	0.86	812
0	8.15	8.35	**93**	0.85	0.89	0
0	7.75	7.9	**93.5**	0.93	0.97	0
0	7.3	7.45	**94**	1.25	1.27	0
0	6.9	7.05	**94.5**	1.36	1.38	0
760	6.65	6.7	**95**	1.39	1.4	1179
0	6.1	6.25	**95.5**	1.49	1.53	0
0	5.75	5.85	**96**	1.89	1.94	0
0	5.35	5.5	**96.5**	1.96	2.01	0
0	5	5.1	**97**	2.03	2.07	0
494	4.7	4.75	**97.5**	2.11	2.13	1655
0	4.35	4.5	**98**	2.24	2.29	0
0	4	4.15	**98.5**	2.41	2.46	0
0	3.7	3.8	**99**	2.6	2.7	0
0	3.35	3.45	**99.5**	2.75	2.81	0

3053	3.2	3.3	**100**	2.82	2.86	3157
			100.53			
0	2.55	2.6	**101**	2.97	3.05	0
0	2.1	2.16	**102**	3.5	3.6	0
0	1.67	1.7	**103**	4.05	4.2	0
0	1.08	1.12	**104**	4.45	4.6	0
6764	1.28	1.3	**105**	5.65	5.75	464
0	0.6	0.63	**106**	5.95	6.15	0
0	0.43	0.47	**107**	6.8	6.95	0
0	0.3	0.34	**108**	7.65	7.85	0
0	0.22	0.25	**109**	8.45	8.8	0
4257	0.37	0.39	**110**	9.7	9.85	720
2868	0.11	0.12	**115**	14.45	14.6	587
452	0.04	0.05	**120**	19.35	19.65	120
92	0.02	0.04	**125**	24.3	24.65	6
1032	0.01	0.03	**130**	29.4	29.65	2
1005	0.01	0.02	**135**	34.3	34.6	90
2	0.01	0.03	**140**	39.4	39.65	11
2	0.01	0.02	**145**	44.35	44.65	3
3	0.01	0.03	**150**	49.3	49.6	7
11	0.02	0.02	**155**	54.3	54.6	285
2	0.02	0.02	**160**	58	59.65	6
3	0.01	0.02	**165**	64.3	64.6	10
14	0.01	0.02	**170**	69.3	69.6	10
7	0.01	0.02	**175**	73	74.65	80
50	0.01	0.02	**180**	79.3	79.6	500
1	0.01	0.02	**185**	83.05	84.65	500
1	0.01	0.02	**190**	89.3	89.6	20
202	0.01	0.02	**195**	94.45	94.7	2

expiration date: 20 May, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
7	50.45	50.85	**50**	0.03	0.04	35
0	45.4	45.7	**55**	0.04	0.05	3
1	40.45	40.75	**60**	0.07	0.08	404
500	35.5	35.8	**65**	0.12	0.13	400
102	30.55	30.85	**70**	0.19	0.21	32
20	25.7	26.05	**75**	0.31	0.33	123
6	20.85	21.1	**80**	0.53	0.55	288
36	16.3	16.5	**85**	0.94	0.95	3832
7	14.05	14.3	**87.5**	1.24	1.27	72
89	12	12.15	**90**	1.64	1.68	812
14	9.95	10.15	**92.5**	2.15	2.22	164
84	8.2	8.3	**95**	2.87	2.91	630
113	6.45	6.55	**97.5**	3.8	3.85	208

1353	5.05	5.2	**100**	4.8	4.9	1928
			100.53			
4407	2.8	2.82	**105**	7.55	7.65	19
1365	1.37	1.39	**110**	11.15	11.3	49
2088	0.62	0.65	**115**	15.3	15.6	12
515	0.28	0.3	**120**	20.05	20.35	31
799	0.14	0.15	**125**	24.8	25.2	77
50	0.07	0.09	**130**	29.8	30.15	3
22	0.04	0.09	**135**	34.7	35.1	2
1	0.01	0.07	**140**	39.75	40.15	4
100	0.01	0.06	**145**	44.7	45.1	20

expiration date: 17 June, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
	90.3	90.75	**10**	0	0.03	
	86.6	89.8	**12.5**	0	0.03	
	85.35	85.8	**15**	0	0.03	
	82.9	83.35	**17.5**	0	0.03	
	80.4	80.85	**20**	0	0.03	
	77.9	78.35	**22.5**	0	0.03	
	75.4	75.85	**25**	0	0.03	
	70.25	70.65	**30**	0	0.04	
	65.25	65.65	**35**	0.01	0.04	
	60.4	60.7	**40**	0.01	0.05	
	55.4	55.85	**45**	0.01	0.06	60
	50.35	50.8	**50**	0.03	0.08	20
3	45.4	45.7	**55**	0.07	0.11	8
4	40.5	40.95	**60**	0.11	0.15	6
1	35.55	35.95	**65**	0.18	0.24	32
20	30.6	30.85	**70**	0.29	0.34	1
1	25.75	26	**75**	0.48	0.52	6
3	21.05	21.25	**80**	0.76	0.82	853
3	16.6	16.8	**85**	1.3	1.34	30
51	14.4	14.55	**87.5**	1.67	1.73	441
9	12.4	12.6	**90**	2.18	2.19	18
61	10.5	10.65	**92.5**	2.72	2.79	9
230	8.75	8.95	**95**	3.5	3.55	290
64	7.1	7.2	**97.5**	4.4	4.5	52
607	5.65	5.75	**100**	5.45	5.55	164
			100.53			
680	3.45	3.55	**105**	8.15	8.3	236
739	1.9	1.97	**110**	11.65	11.8	630
18396	0.92	1	**115**	15.7	15.9	908
1375	0.45	0.5	**120**	20.2	20.5	40
707	0.24	0.28	**125**	24.95	25.3	548

1874	0.12	0.17	**130**	29.95	30.2	5
51	0.07	0.1	**135**	34.8	35.15	7
41	0.06	0.07	**140**	39.65	40.1	101
6	0.02	0.08	**145**	44.65	45.1	14
54	0.03	0.07	**150**	49.6	50.05	1
1	0.02	0.06	**155**	54.7	55.05	10
80	0.01	0.06	**160**	59.65	60	1
1	0.01	0.05	**165**	64.6	65.05	10
10	0.01	0.05	**170**	68.4	70.35	12
1	0.01	0.05	**175**	74.55	75	100
51	0.01	0.04	**180**	79.55	80	7
152	0.01	0.04	**185**	84.55	85	0
10	0.01	0.04	**190**	89.55	90	0
21	0.01	0.04	**195**	94.7	95	40

expiration date: 15 July, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
20	50.4	50.7	**50**	0.06	0.11	6
22	45.45	45.7	**55**	0.11	0.16	10
30	39.35	40.8	**60**	0.18	0.23	250
1	35.6	35.85	**65**	0.27	0.32	25
1	30.75	31	**70**	0.41	0.47	14
2	25.95	26.2	**75**	0.64	0.75	12
9	21.35	21.6	**80**	1.01	1.08	166
7	16.95	17.1	**85**	1.62	1.68	159
15	14.85	15	**87.5**	2.08	2.14	50
37	12.9	13.1	**90**	2.6	2.65	188
1	11.05	11.2	**92.5**	3.25	3.35	52
161	9.3	9.45	**95**	4.05	4.15	23
36	7.75	7.9	**97.5**	4.95	5.05	614
208	6.35	6.45	**100**	6.05	6.2	126
			100.53			
262	4.05	4.15	**105**	8.6	8.85	55
430	2.43	2.49	**110**	12.1	12.25	47
122	1.36	1.42	**115**	16.1	16.3	67
422	0.72	0.78	**120**	20.5	20.75	5
34	0.38	0.43	**125**	25	25.35	59
51	0.14	0.23	**130**	29.95	30.25	4
5	0.11	0.15	**135**	34.85	35.2	4
22	0.07	0.12	**140**	39.7	40.15	4
5	0.04	0.09	**145**	44.8	45.15	30
11	0.02	0.09	**150**	49.6	50.05	1
10	0.01	0.07	**155**	54.6	55.05	96
2	0.01	0.05	**160**	59.7	60.05	10
6	0.01	0.06	**165**	64.65	65.05	10

20	0.01	0.05	**170**	69.75	70.2	19
200	0.01	0.05	**175**	74.7	75.1	93
10	0.01	0.05	**180**	79.75	80.15	90

expiration date: 21 October, 2016						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
7	50.4	50.85	**50**	0.24	0.29	100
7	45.45	45.95	**55**	0.34	0.4	10
1	40.55	40.9	**60**	0.5	0.56	3
10	35.8	36.15	**65**	0.71	0.78	20
7	31.05	31.4	**70**	1.05	1.12	2
10	26.55	26.8	**75**	1.53	1.6	5
16	22.2	22.35	**80**	2.25	2.31	55
5	18.25	18.4	**85**	3.2	3.3	154
1	16.3	16.5	**87.5**	3.85	5.3	71
86	14.5	14.65	**90**	4.55	4.65	85
17	12.8	13	**92.5**	5.35	5.45	25
74	11.25	11.4	**95**	6.25	6.4	38
10	9.85	9.95	**97.5**	7.3	7.45	32
82	8.5	8.65	**100**	8.45	8.6	644
			100.53			
184	6.15	6.3	**105**	11.15	11.3	70
294	4.4	4.5	**110**	14.35	14.5	605
158	3	3.1	**115**	17.95	18.2	84
867	2.02	2.1	**120**	21.95	22.25	10
365	1.35	1.42	**125**	26.3	26.55	77
862	0.89	0.96	**130**	30.8	31.1	5
230	0.58	0.65	**135**	35.5	35.85	11
127	0.38	0.44	**140**	40.15	40.65	1
40	0.25	0.3	**145**	45.15	45.5	1
5	0.19	0.22	**150**	50	50.45	1
12	0.11	0.17	**155**	54.95	55.35	0
1	0.07	0.14	**160**	59.9	60.3	10
0	0.05	0.12	**165**	64.85	65.25	0
0	0.03	0.1	**170**	69.85	70.4	0
0	0.02	0.09	**175**	74.8	75.3	0
0	0.01	0.08	**180**	79.8	80.3	0
0	0.01	0.07	**185**	84.75	85.25	0
0	0	0.07	**190**	89.75	91	0
0	0	0.06	**195**	94.7	95.15	0

expiration date: 20 January, 2017						
volume	Bid	Ask	**Strike**	Bid	Ask	volume

1	52.5	53	**47.5**	0.47	0.48	236
22	49.95	50.55	**50**	0.55	0.56	60
2	45.15	45.65	**55**	0.75	0.77	104
1	40.4	40.85	**60**	1.01	1.04	101
1	35.6	36.1	**65**	1.39	1.41	29
18	31.2	31.55	**70**	1.9	1.94	154
33	26.9	27.2	**75**	2.61	2.65	246
144	22.75	23.1	**80**	3.55	3.6	415
36	19.05	19.25	**85**	4.75	4.85	189
1	17.25	17.55	**87.5**	5.5	5.6	51
69	15.6	15.8	**90**	6.35	6.45	313
12	14.05	14.2	**92.5**	7.25	7.35	83
75	12.55	12.75	**95**	8.3	8.4	1014
60	11.2	11.35	**97.5**	9.4	9.55	77
671	9.95	10.05	**100**	10.65	10.75	850
			100.53			
186	7.7	7.8	**105**	13.35	13.5	155
484	5.85	5.95	**110**	16.5	16.65	165
904	4.35	4.45	**115**	20	20.15	2981
824	3.2	3.25	**120**	23.8	24	183
593	2.31	2.36	**125**	27.9	28.15	5
235	1.65	1.69	**130**	32.25	32.45	111
37	1.19	1.22	**135**	36.75	36.95	1
94	0.85	0.88	**140**	41.3	41.7	10
37	0.62	0.64	**145**	46.15	46.35	3
126	0.45	0.47	**150**	50.95	51.2	2
39	0.33	0.35	**155**	55.7	56.1	14
29	0.25	0.26	**160**	60.55	61.05	5
3	0.19	0.2	**165**	65.45	65.9	70
4	0.15	0.16	**170**	70.4	70.85	1
9	0.12	0.13	**175**	75.35	75.8	3
10	0.1	0.12	**180**	80.35	80.7	1
1	0.08	0.1	**185**	85.15	85.8	4
10	0.06	0.1	**190**	90.1	90.8	42
17	0.06	0.09	**195**	95.25	95.6	3

expiration date: 16 June, 2017						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
0	52.05	54.2	**47.5**	0.73	0.92	10
1	49.85	51.8	**50**	0.9	1.07	1
1	45.05	46.9	**55**	1.19	1.44	5
2	40.7	42.1	**60**	1.69	1.93	6
20	36.2	37.45	**65**	2.33	2.53	10
13	32	33	**70**	3.1	3.35	3
5	27.95	29.05	**75**	3.9	4.25	6

84	24.35	24.95	**80**	5.3	5.55	25
25	20.85	21.5	**85**	6.75	7.05	1
1	19.25	19.85	**87.5**	7.65	7.85	5
252	17.7	18.25	**90**	8.6	8.8	10
2	16.25	16.65	**92.5**	9.55	9.85	112
10	14.85	15.2	**95**	10.7	10.9	1
3	13.55	13.9	**97.5**	11.85	12.15	220
15	12.45	12.7	**100**	13.1	13.45	548
			100.53			
5	10.2	10.45	**105**	15.75	16.2	50
49	8.2	8.5	**110**	18.8	19.35	29
31	6.65	6.9	**115**	22.15	22.65	1
63	5.2	5.5	**120**	25.75	26.35	16
2	4.15	4.4	**125**	29.6	30.25	1
24	3.2	3.55	**130**	33.65	34.25	199
8	2.58	2.75	**135**	37.6	38.7	100
8	1.98	2.18	**140**	41.95	43.25	100
15	1.56	1.73	**145**	46.3	47.7	0

expiration date: 19 January, 2018						
volume	Bid	Ask	**Strike**	Bid	Ask	volume
11	51.65	53.85	**47.5**	1.34	1.65	10
1	48	51.15	**50**	1.55	1.85	6
1	45.05	47.4	**55**	2.15	2.45	48
18	40.8	42.9	**60**	2.87	3.25	35
1	36.85	38.3	**65**	3.7	3.95	89
63	33.35	34.5	**70**	4.75	5	130
56	29.7	30.5	**75**	6.05	6.3	216
8	26.4	26.9	**80**	7.55	7.8	24
4	23.1	23.8	**85**	9.3	9.55	22
1	21.55	22.3	**87.5**	10.2	10.5	2
18	20.25	20.75	**90**	11.3	11.55	888
6	18.8	19.45	**92.5**	12.3	12.65	177
47	17.55	18.05	**95**	13.45	13.75	15
20	16.3	16.8	**97.5**	14.7	15	175
299	15.3	15.8	**100**	16	16.2	66
			100.53			
59	13	13.45	**105**	18.75	19.2	501
26	11.3	11.55	**110**	21.85	22.15	12
97	9.45	9.85	**115**	25.05	25.45	162
488	8	8.4	**120**	28.5	28.85	1
8	6.75	7.1	**125**	32.15	32.8	1
59	5.7	6	**130**	35.9	36.45	2
4	4.75	5.05	**135**	39.9	40.45	2
92	4.05	4.15	**140**	44	44.75	72

22	3.35	3.6	**145**	48.3	49	10
85	2.9	3.05	**150**	52.4	53.5	406
12	2.29	2.61	**155**	57.05	58.1	14
101	1.9	2.15	**160**	61.5	62.6	1
1	1.65	1.85	**165**	65.8	67.3	17
210	1.28	1.58	**170**	70	71.45	6
278	0.98	1.33	**175**	75	76.45	278
11	0.9	1.15	**180**	80	81.35	8

Source: Chicago Board Options Exchange.

Annex 2: Implied volatility matrix for bid price

Time to maturity		17	33	44	68	79	95	106	123
K	K/S0	0.067	0.131	0.175	0.270	0.313	0.377	0.421	0.488
45	0.448	44.26%	42.09%	40.60%	49.08%	52.97%	51.61%	50.67%	46.57%
47.5	0.472	44.71%	42.39%	40.79%	49.52%	53.52%	50.23%	47.97%	45.67%
50	0.497	45.17%	42.68%	40.96%	49.95%	54.07%	48.78%	45.14%	44.72%
55	0.547	46.03%	43.20%	41.25%	44.86%	46.51%	44.13%	42.50%	41.57%
60	0.597	46.81%	43.64%	41.46%	42.46%	42.92%	40.98%	39.65%	37.62%
65	0.647	47.53%	44.00%	41.58%	39.93%	39.17%	37.29%	36.00%	34.93%
70	0.696	48.17%	44.29%	41.62%	37.20%	35.18%	33.53%	32.40%	31.80%
75	0.746	48.74%	40.97%	35.63%	33.30%	32.23%	30.74%	29.72%	29.16%
76	0.756	48.85%	40.46%	34.70%	32.56%	31.57%	30.20%	29.26%	28.73%
77	0.766	44.90%	38.30%	33.76%	31.81%	30.92%	29.66%	28.79%	28.31%
78	0.776	45.13%	37.84%	32.83%	31.08%	30.27%	29.12%	28.33%	27.88%
79	0.786	44.39%	36.98%	31.89%	30.34%	29.62%	28.59%	27.87%	27.46%
80	0.796	41.39%	35.20%	30.95%	29.60%	28.98%	28.05%	27.41%	27.04%
81	0.806	35.84%	32.21%	29.72%	28.94%	28.58%	27.69%	27.08%	26.69%
82	0.816	39.49%	32.97%	28.49%	28.28%	28.19%	27.34%	26.76%	26.35%
83	0.826	37.64%	31.49%	27.26%	27.63%	27.80%	26.99%	26.43%	26.01%
84	0.836	35.81%	30.02%	26.03%	26.98%	27.41%	26.64%	26.11%	25.67%
85	0.846	35.72%	30.16%	26.33%	26.81%	27.03%	26.29%	25.79%	25.33%
85.5	0.850	34.75%	28.07%	23.47%	25.76%	26.81%	26.10%	25.61%	25.18%
86	0.855	36.15%	28.84%	23.82%	25.69%	26.55%	25.86%	25.39%	25.00%
86.5	0.860	35.13%	28.41%	23.79%	25.50%	26.29%	25.62%	25.17%	24.82%
87	0.865	34.34%	27.78%	23.27%	25.16%	26.03%	25.38%	24.94%	24.64%
87.5	0.870	33.53%	28.35%	24.78%	25.46%	25.77%	25.15%	24.72%	24.46%
88	0.875	33.48%	27.19%	22.87%	24.75%	25.61%	25.01%	24.60%	24.33%
88.5	0.880	30.99%	26.09%	22.73%	24.60%	25.45%	24.88%	24.48%	24.20%
89	0.885	30.14%	25.68%	22.61%	24.45%	25.30%	24.74%	24.36%	24.07%
89.5	0.890	30.24%	25.57%	22.36%	24.27%	25.14%	24.61%	24.25%	23.95%
90	0.895	29.34%	25.83%	23.42%	24.50%	24.99%	24.48%	24.13%	23.82%
90.5	0.900	29.36%	24.54%	21.23%	23.65%	24.77%	24.29%	23.96%	23.68%
91	0.905	28.50%	24.19%	21.22%	23.50%	24.54%	24.10%	23.79%	23.53%
91.5	0.910	28.22%	23.87%	20.88%	23.24%	24.32%	23.91%	23.63%	23.39%
92	0.915	27.94%	23.56%	20.55%	22.98%	24.10%	23.72%	23.46%	23.25%
92.5	0.920	27.66%	24.17%	21.78%	23.22%	23.87%	23.53%	23.30%	23.11%
93	0.925	27.63%	23.32%	20.36%	22.71%	23.78%	23.43%	23.19%	22.99%
93.5	0.930	27.21%	23.08%	20.24%	22.61%	23.69%	23.33%	23.09%	22.87%
94	0.935	26.78%	23.25%	20.83%	22.73%	23.60%	23.23%	22.98%	22.76%
94.5	0.940	25.50%	22.64%	20.67%	22.62%	23.51%	23.13%	22.88%	22.64%
95	0.945	25.93%	22.85%	20.73%	22.57%	23.42%	23.04%	22.77%	22.53%
95.5	0.950	25.74%	22.25%	19.86%	22.19%	23.26%	22.89%	22.63%	22.40%
96	0.955	25.05%	22.48%	20.72%	22.36%	23.11%	22.75%	22.50%	22.28%
96.5	0.960	24.28%	21.85%	20.17%	22.08%	22.96%	22.60%	22.36%	22.16%

97	0.965	24.25%	21.58%	19.75%	21.85%	22.81%	22.46%	22.22%	22.04%
97.5	0.970	23.96%	21.30%	19.47%	21.66%	22.66%	22.32%	22.08%	21.92%
98	0.975	23.42%	20.88%	19.12%	21.46%	22.53%	22.19%	21.96%	21.81%
98.5	0.980	22.85%	20.47%	18.84%	21.28%	22.40%	22.06%	21.83%	21.70%
99	0.985	22.89%	20.41%	18.70%	21.15%	22.28%	21.94%	21.70%	21.60%
99.5	0.990	22.27%	19.86%	18.21%	20.92%	22.16%	21.81%	21.58%	21.49%
100	0.995	22.26%	19.75%	18.02%	20.77%	22.03%	21.69%	21.45%	21.39%
100.53	1.000	22.05%	19.17%	17.20%	20.44%	21.93%	21.60%	21.37%	21.28%
101	1.005	21.84%	18.60%	16.38%	20.11%	21.82%	21.51%	21.29%	21.18%
102	1.015	21.18%	18.19%	16.12%	19.89%	21.61%	21.33%	21.13%	20.99%
103	1.025	20.54%	17.63%	15.63%	19.59%	21.41%	21.15%	20.97%	20.79%
104	1.035	19.79%	16.11%	13.59%	18.81%	21.21%	20.98%	20.82%	20.60%
105	1.044	18.98%	17.71%	16.83%	19.71%	21.04%	20.83%	20.69%	20.43%
106	1.054	17.65%	14.73%	12.72%	18.33%	20.90%	20.70%	20.57%	20.33%
107	1.064	17.65%	14.80%	12.84%	18.27%	20.77%	20.58%	20.45%	20.24%
108	1.074	17.65%	14.86%	12.95%	18.22%	20.64%	20.46%	20.34%	20.16%
109	1.084	17.65%	14.93%	13.06%	18.17%	20.51%	20.35%	20.23%	20.07%
110	1.094	17.65%	15.00%	13.18%	18.12%	20.38%	20.23%	20.12%	19.98%
111	1.104	17.66%	15.08%	13.30%	18.08%	20.26%	20.11%	20.00%	19.90%
112	1.114	17.67%	15.16%	13.43%	18.03%	20.14%	19.99%	19.88%	19.81%
113	1.124	17.69%	15.24%	13.56%	17.99%	20.02%	19.87%	19.76%	19.73%
114	1.134	17.70%	15.32%	13.68%	17.95%	19.90%	19.74%	19.64%	19.64%
115	1.144	17.71%	15.40%	13.81%	17.90%	19.78%	19.62%	19.52%	19.55%
116	1.154	17.75%	15.50%	13.95%	18.06%	19.94%	19.68%	19.51%	19.53%
117	1.164	17.78%	15.59%	14.09%	18.21%	20.10%	19.74%	19.50%	19.51%
118	1.174	17.81%	15.69%	14.23%	18.37%	20.26%	19.80%	19.49%	19.49%
119	1.184	17.84%	15.78%	14.37%	18.52%	20.42%	19.86%	19.48%	19.47%
120	1.194	17.87%	15.88%	14.51%	18.67%	20.58%	19.92%	19.47%	19.45%
125	1.243	18.11%	16.42%	15.26%	18.83%	20.46%	20.11%	19.86%	19.14%
130	1.293	18.45%	17.06%	16.10%	20.33%	22.27%	21.77%	21.42%	20.02%
135	1.343	18.88%	17.76%	16.99%	20.39%	21.94%	21.75%	21.61%	20.73%
140	1.393	19.41%	18.55%	17.96%	22.15%	24.07%	22.40%	21.25%	19.84%
145	1.442	20.02%	19.39%	18.96%	23.00%	24.86%	23.35%	22.31%	22.27%
150	1.492	20.74%	20.33%	20.04%	24.02%	25.85%	24.39%	23.39%	23.34%
155	1.542	21.57%	21.34%	21.19%	25.05%	26.82%	25.35%	24.34%	24.35%
160	1.592	22.49%	22.43%	22.39%	25.94%	27.57%	26.18%	25.22%	25.24%
165	1.641	23.49%	23.57%	23.62%	26.77%	28.22%	26.89%	25.98%	26.01%
170	1.691	24.63%	24.81%	24.94%	27.55%	28.75%	27.50%	26.64%	26.69%
175	1.741	25.86%	26.05%	26.18%	28.23%	29.16%	27.99%	27.19%	27.27%
180	1.791	27.08%	27.15%	27.20%	28.76%	29.48%	28.39%	27.65%	27.76%
185	1.840	27.93%	27.96%	27.98%	29.16%	29.70%	28.71%	28.04%	28.16%
190	1.890	28.64%	28.59%	28.55%	29.44%	29.85%	28.97%	28.37%	28.52%
195	1.940	29.10%	29.04%	29.00%	29.65%	29.94%	29.17%	28.64%	28.81%

Time to maturity		134	157	184	230	319	465	678
K	K/S0	0.532	0.623	0.730	0.913	1.266	1.845	2.690
45	0.448	43.92%	43.03%	41.99%	40.22%	40.97%	35.11%	33.05%

47.5	0.472	44.19%	43.28%	42.22%	40.41%	41.18%	35.20%	33.07%
50	0.497	44.44%	43.52%	42.44%	40.59%	38.90%	34.85%	27.08%
55	0.547	40.96%	39.98%	38.82%	36.85%	36.00%	32.15%	30.97%
60	0.597	36.30%	35.70%	35.00%	33.81%	33.37%	31.14%	29.78%
65	0.647	34.24%	33.59%	32.82%	31.50%	30.78%	29.55%	28.87%
70	0.696	31.41%	30.90%	30.29%	29.26%	29.27%	28.35%	28.43%
75	0.746	28.79%	28.52%	28.19%	27.65%	27.81%	26.97%	27.63%
76	0.756	28.39%	28.14%	27.85%	27.36%	27.53%	26.89%	27.51%
77	0.766	27.99%	27.77%	27.51%	27.07%	27.25%	26.81%	27.39%
78	0.776	27.59%	27.40%	27.17%	26.78%	26.97%	26.72%	27.27%
79	0.786	27.19%	27.02%	26.83%	26.49%	26.69%	26.64%	27.15%
80	0.796	26.80%	26.65%	26.49%	26.20%	26.41%	26.55%	27.02%
81	0.806	26.44%	26.33%	26.20%	25.98%	26.21%	26.38%	26.88%
82	0.816	26.09%	26.01%	25.92%	25.76%	26.01%	26.21%	26.72%
83	0.826	25.73%	25.69%	25.63%	25.54%	25.82%	26.03%	26.57%
84	0.836	25.38%	25.37%	25.35%	25.32%	25.63%	25.86%	26.42%
85	0.846	25.03%	25.05%	25.07%	25.10%	25.44%	25.69%	26.26%
85.5	0.850	24.91%	24.93%	24.96%	25.00%	25.35%	25.64%	26.20%
86	0.855	24.75%	24.78%	24.82%	24.88%	25.25%	25.58%	26.11%
86.5	0.860	24.60%	24.64%	24.68%	24.76%	25.15%	25.53%	26.03%
87	0.865	24.44%	24.49%	24.55%	24.64%	25.04%	25.47%	25.95%
87.5	0.870	24.29%	24.34%	24.41%	24.52%	24.94%	25.41%	25.86%
88	0.875	24.15%	24.22%	24.29%	24.41%	24.87%	25.35%	25.84%
88.5	0.880	24.02%	24.09%	24.17%	24.31%	24.79%	25.29%	25.83%
89	0.885	23.88%	23.96%	24.05%	24.20%	24.72%	25.23%	25.81%
89.5	0.890	23.75%	23.83%	23.93%	24.10%	24.64%	25.16%	25.79%
90	0.895	23.62%	23.71%	23.81%	23.99%	24.57%	25.10%	25.77%
90.5	0.900	23.49%	23.59%	23.70%	23.89%	24.50%	25.03%	25.68%
91	0.905	23.36%	23.47%	23.59%	23.79%	24.42%	24.96%	25.60%
91.5	0.910	23.24%	23.35%	23.47%	23.69%	24.34%	24.89%	25.52%
92	0.915	23.11%	23.23%	23.36%	23.59%	24.26%	24.82%	25.44%
92.5	0.920	22.98%	23.11%	23.25%	23.49%	24.18%	24.75%	25.35%
93	0.925	22.86%	22.99%	23.15%	23.41%	24.11%	24.71%	25.31%
93.5	0.930	22.74%	22.88%	23.04%	23.32%	24.04%	24.66%	25.27%
94	0.935	22.61%	22.76%	22.94%	23.24%	23.97%	24.61%	25.23%
94.5	0.940	22.49%	22.65%	22.84%	23.16%	23.90%	24.56%	25.19%
95	0.945	22.37%	22.54%	22.73%	23.07%	23.83%	24.52%	25.15%
95.5	0.950	22.25%	22.43%	22.65%	23.01%	23.76%	24.46%	25.11%
96	0.955	22.14%	22.33%	22.56%	22.95%	23.70%	24.40%	25.06%
96.5	0.960	22.03%	22.23%	22.48%	22.89%	23.63%	24.35%	25.02%
97	0.965	21.92%	22.14%	22.39%	22.83%	23.57%	24.29%	24.98%
97.5	0.970	21.81%	22.04%	22.31%	22.77%	23.50%	24.24%	24.93%
98	0.975	21.71%	21.95%	22.22%	22.69%	23.45%	24.21%	24.92%
98.5	0.980	21.62%	21.86%	22.14%	22.61%	23.40%	24.18%	24.91%
99	0.985	21.53%	21.77%	22.05%	22.53%	23.36%	24.16%	24.89%
99.5	0.990	21.43%	21.68%	21.96%	22.45%	23.31%	24.13%	24.88%
100	0.995	21.34%	21.59%	21.88%	22.37%	23.26%	24.11%	24.87%

100.53	1.000	21.23%	21.49%	21.79%	22.30%	23.19%	24.05%	24.81%
101	1.005	21.12%	21.38%	21.70%	22.23%	23.13%	23.99%	24.76%
102	1.015	20.89%	21.18%	21.52%	22.09%	23.01%	23.87%	24.66%
103	1.025	20.67%	20.98%	21.34%	21.95%	22.89%	23.76%	24.55%
104	1.035	20.45%	20.78%	21.16%	21.81%	22.76%	23.64%	24.44%
105	1.044	20.26%	20.60%	21.00%	21.68%	22.66%	23.54%	24.35%
106	1.054	20.18%	20.52%	20.92%	21.60%	22.57%	23.44%	24.33%
107	1.064	20.11%	20.45%	20.84%	21.52%	22.48%	23.34%	24.31%
108	1.074	20.04%	20.37%	20.77%	21.44%	22.39%	23.25%	24.29%
109	1.084	19.96%	20.30%	20.69%	21.37%	22.30%	23.15%	24.27%
110	1.094	19.89%	20.23%	20.62%	21.29%	22.22%	23.06%	24.24%
111	1.104	19.83%	20.16%	20.54%	21.19%	22.14%	23.00%	24.15%
112	1.114	19.77%	20.08%	20.46%	21.10%	22.06%	22.95%	24.06%
113	1.124	19.70%	20.01%	20.38%	21.00%	21.98%	22.90%	23.97%
114	1.134	19.64%	19.94%	20.30%	20.91%	21.90%	22.85%	23.87%
115	1.144	19.58%	19.87%	20.22%	20.81%	21.82%	22.79%	23.78%
116	1.154	19.55%	19.84%	20.18%	20.76%	21.76%	22.71%	23.73%
117	1.164	19.52%	19.80%	20.14%	20.70%	21.69%	22.63%	23.68%
118	1.174	19.49%	19.77%	20.09%	20.65%	21.63%	22.55%	23.63%
119	1.184	19.46%	19.73%	20.05%	20.60%	21.57%	22.47%	23.58%
120	1.194	19.43%	19.70%	20.01%	20.54%	21.51%	22.39%	23.53%
125	1.243	18.68%	19.12%	19.64%	20.52%	21.27%	22.22%	23.32%
130	1.293	19.12%	19.43%	19.80%	20.43%	21.17%	21.97%	23.10%
135	1.343	20.15%	20.25%	20.35%	20.54%	21.15%	21.57%	22.93%
140	1.393	18.93%	19.19%	19.50%	20.03%	20.97%	21.34%	22.87%
145	1.442	22.25%	21.95%	21.60%	21.00%	21.46%	20.98%	22.80%
150	1.492	23.30%	22.79%	22.20%	21.20%	21.67%	21.86%	22.57%
155	1.542	24.35%	23.67%	22.87%	21.51%	21.54%	22.69%	22.61%
160	1.592	25.26%	24.46%	23.51%	21.90%	21.68%	23.43%	22.51%
165	1.641	26.03%	25.15%	24.12%	22.36%	21.97%	24.12%	22.19%
170	1.691	26.73%	25.87%	24.86%	23.13%	22.56%	24.76%	21.13%
175	1.741	27.32%	26.39%	25.29%	23.42%	23.13%	25.34%	21.62%
180	1.791	27.83%	26.92%	25.85%	24.03%	24.08%	25.88%	22.50%
185	1.840	28.25%	27.38%	26.35%	24.60%	23.21%	26.38%	23.18%
190	1.890	28.61%	27.80%	26.84%	25.21%	23.51%	26.85%	23.83%
195	1.940	28.92%	28.16%	27.28%	25.77%	26.10%	27.28%	24.45%

Source: Own calculation.

Annex 3: Implied volatility matrix for ask price

Time to maturity		17	33	44	68	79	95	106	123
K	K/S0	0.067	0.131	0.175	0.270	0.313	0.377	0.421	0.488
45	0.448	58.92%	63.09%	65.95%	64.27%	63.50%	64.15%	64.59%	56.06%
47.5	0.472	58.89%	63.05%	65.92%	64.24%	63.47%	61.95%	60.91%	54.60%
50	0.497	58.85%	63.02%	65.88%	64.20%	63.43%	59.76%	57.24%	53.13%
55	0.547	58.78%	62.95%	65.81%	58.05%	54.50%	51.95%	50.19%	47.22%
60	0.597	58.72%	59.27%	59.65%	52.48%	49.20%	47.65%	46.58%	43.37%
65	0.647	58.65%	55.34%	53.07%	47.00%	44.22%	42.70%	41.66%	38.82%
70	0.696	58.58%	50.80%	45.45%	41.33%	39.45%	37.44%	36.06%	34.57%
75	0.746	58.51%	46.50%	38.24%	36.49%	35.69%	33.75%	32.42%	31.39%
76	0.756	58.50%	45.87%	37.18%	35.53%	34.78%	33.00%	31.79%	30.82%
77	0.766	53.54%	43.22%	36.12%	34.58%	33.87%	32.26%	31.15%	30.26%
78	0.776	54.15%	42.84%	35.06%	33.62%	32.96%	31.51%	30.52%	29.69%
79	0.786	51.15%	40.98%	34.00%	32.66%	32.05%	30.77%	29.88%	29.12%
80	0.796	50.06%	39.91%	32.94%	31.70%	31.14%	30.02%	29.25%	28.55%
81	0.806	45.70%	37.46%	31.79%	30.95%	30.56%	29.52%	28.80%	28.08%
82	0.816	46.29%	37.02%	30.64%	30.19%	29.99%	29.02%	28.35%	27.62%
83	0.826	44.62%	35.66%	29.49%	29.44%	29.41%	28.52%	27.91%	27.15%
84	0.836	41.81%	33.83%	28.35%	28.68%	28.84%	28.02%	27.46%	26.69%
85	0.846	41.34%	33.30%	27.77%	28.11%	28.26%	27.52%	27.01%	26.22%
85.5	0.850	40.98%	32.01%	25.84%	27.34%	28.02%	27.27%	26.74%	26.02%
86	0.855	41.96%	32.52%	26.03%	27.23%	27.78%	27.01%	26.48%	25.81%
86.5	0.860	39.66%	31.25%	25.47%	26.89%	27.54%	26.75%	26.21%	25.61%
87	0.865	38.75%	30.96%	25.61%	26.77%	27.30%	26.50%	25.94%	25.40%
87.5	0.870	38.79%	31.17%	25.93%	26.71%	27.06%	26.24%	25.68%	25.20%
88	0.875	37.60%	30.33%	25.34%	26.35%	26.81%	26.05%	25.53%	25.06%
88.5	0.880	35.48%	29.13%	24.77%	26.00%	26.56%	25.86%	25.38%	24.92%
89	0.885	35.09%	28.41%	23.82%	25.52%	26.31%	25.67%	25.23%	24.78%
89.5	0.890	34.23%	28.08%	23.86%	25.36%	26.05%	25.47%	25.08%	24.64%
90	0.895	33.33%	27.96%	24.26%	25.32%	25.80%	25.28%	24.93%	24.50%
90.5	0.900	32.93%	26.84%	22.65%	24.67%	25.60%	25.09%	24.74%	24.35%
91	0.905	31.97%	26.24%	22.30%	24.42%	25.39%	24.90%	24.56%	24.19%
91.5	0.910	31.56%	25.85%	21.92%	24.16%	25.19%	24.70%	24.37%	24.03%
92	0.915	30.63%	25.25%	21.56%	23.91%	24.99%	24.51%	24.18%	23.87%
92.5	0.920	30.10%	25.48%	22.31%	24.00%	24.78%	24.32%	24.00%	23.72%
93	0.925	29.98%	24.83%	21.29%	23.56%	24.60%	24.18%	23.89%	23.59%
93.5	0.930	29.44%	24.39%	20.92%	23.32%	24.42%	24.04%	23.77%	23.47%
94	0.935	28.83%	24.43%	21.40%	23.35%	24.24%	23.90%	23.66%	23.34%
94.5	0.940	27.81%	23.91%	21.23%	23.17%	24.06%	23.75%	23.55%	23.21%
95	0.945	27.86%	23.71%	20.85%	22.92%	23.88%	23.61%	23.43%	23.09%
95.5	0.950	27.19%	23.20%	20.46%	22.69%	23.71%	23.44%	23.26%	22.95%
96	0.955	26.75%	23.44%	21.16%	22.80%	23.55%	23.28%	23.08%	22.81%
96.5	0.960	25.73%	22.80%	20.78%	22.57%	23.39%	23.11%	22.91%	22.67%
97	0.965	25.81%	22.46%	20.15%	22.26%	23.23%	22.94%	22.74%	22.52%

97.5	0.970	25.51%	22.03%	19.64%	21.99%	23.07%	22.77%	22.56%	22.38%
98	0.975	24.66%	21.73%	19.71%	21.95%	22.98%	22.65%	22.42%	22.27%
98.5	0.980	25.23%	21.79%	19.42%	21.80%	22.88%	22.53%	22.28%	22.16%
99	0.985	24.04%	21.22%	19.28%	21.69%	22.79%	22.41%	22.14%	22.04%
99.5	0.990	23.28%	20.55%	18.68%	21.44%	22.70%	22.29%	22.00%	21.93%
100	0.995	23.39%	20.45%	18.43%	21.30%	22.61%	22.17%	21.87%	21.81%
100.53	1.000	23.02%	19.78%	17.55%	20.92%	22.47%	22.06%	21.79%	21.72%
101	1.005	22.70%	19.19%	16.77%	20.59%	22.34%	21.97%	21.71%	21.64%
102	1.015	22.11%	18.87%	16.64%	20.36%	22.06%	21.77%	21.56%	21.47%
103	1.025	21.53%	18.40%	16.25%	20.05%	21.79%	21.57%	21.41%	21.30%
104	1.035	20.78%	16.98%	14.36%	19.27%	21.52%	21.36%	21.26%	21.13%
105	1.044	20.55%	18.66%	17.36%	20.03%	21.25%	21.16%	21.11%	20.96%
106	1.054	19.61%	16.33%	14.07%	18.92%	21.14%	21.04%	20.97%	20.83%
107	1.064	19.91%	16.25%	13.74%	18.74%	21.03%	20.91%	20.84%	20.71%
108	1.074	19.01%	15.86%	13.69%	18.64%	20.92%	20.79%	20.70%	20.58%
109	1.084	19.00%	15.98%	13.91%	18.64%	20.81%	20.66%	20.57%	20.46%
110	1.094	18.98%	17.47%	16.43%	19.36%	20.70%	20.54%	20.43%	20.33%
111	1.104	18.97%	17.46%	16.42%	19.37%	20.72%	20.50%	20.35%	20.28%
112	1.114	18.96%	17.44%	16.40%	19.38%	20.74%	20.46%	20.26%	20.24%
113	1.124	18.94%	17.43%	16.39%	19.39%	20.76%	20.42%	20.18%	20.19%
114	1.134	18.93%	17.42%	16.38%	19.40%	20.79%	20.38%	20.09%	20.14%
115	1.144	18.92%	17.40%	16.36%	19.41%	20.81%	20.33%	20.01%	20.09%
116	1.154	18.91%	17.39%	16.35%	19.56%	21.03%	20.47%	20.08%	20.14%
117	1.164	18.89%	17.38%	16.34%	19.70%	21.24%	20.60%	20.16%	20.20%
118	1.174	18.88%	17.37%	16.32%	19.84%	21.46%	20.73%	20.23%	20.25%
119	1.184	18.87%	17.35%	16.31%	19.99%	21.67%	20.86%	20.31%	20.30%
120	1.194	18.85%	17.34%	16.30%	20.13%	21.89%	20.99%	20.38%	20.35%
125	1.243	18.79%	17.27%	16.23%	20.99%	23.18%	22.08%	21.33%	20.80%
130	1.293	18.72%	17.21%	16.16%	22.28%	25.09%	23.61%	22.59%	21.87%
135	1.343	18.66%	17.14%	16.09%	23.95%	27.56%	25.38%	23.88%	23.23%
140	1.393	18.60%	17.07%	16.03%	25.80%	30.27%	27.29%	25.25%	24.70%
145	1.442	18.53%	17.01%	15.96%	27.02%	32.10%	29.47%	27.67%	26.64%
150	1.492	18.47%	16.94%	15.89%	26.95%	32.02%	30.24%	29.02%	27.92%
155	1.542	18.40%	16.87%	15.82%	26.88%	31.94%	31.24%	30.75%	29.48%
160	1.592	18.34%	16.81%	15.76%	26.80%	31.87%	31.95%	32.01%	30.72%
165	1.641	18.27%	16.74%	15.69%	26.73%	31.79%	33.20%	34.17%	32.77%
170	1.691	18.21%	16.67%	15.62%	26.66%	31.72%	35.91%	38.79%	36.08%
175	1.741	18.14%	16.61%	15.55%	26.58%	31.64%	34.78%	36.95%	35.87%
180	1.791	18.08%	16.54%	15.48%	26.51%	31.56%	35.47%	38.16%	37.50%
185	1.840	18.02%	16.48%	15.42%	26.44%	31.49%	36.37%	39.73%	38.09%
190	1.890	17.95%	16.41%	15.35%	26.36%	31.41%	37.25%	41.26%	38.66%
195	1.940	17.89%	16.34%	15.28%	26.29%	31.34%	38.09%	42.74%	39.21%

Time to maturity		134	157	184	230	319	465	678
K	K/S0	0.532	0.623	0.730	0.913	1.266	1.845	2.690
45	0.448	50.54%	49.20%	47.62%	44.94%	43.72%	42.39%	38.93%
47.5	0.472	50.51%	49.17%	47.60%	44.92%	43.71%	42.39%	38.93%

50	0.497	50.48%	49.14%	47.58%	44.91%	41.75%	40.69%	36.67%
55	0.547	45.30%	44.24%	43.01%	40.90%	38.14%	37.30%	35.79%
60	0.597	41.28%	40.17%	38.85%	36.62%	35.08%	34.45%	33.69%
65	0.647	36.99%	36.22%	35.32%	33.79%	32.35%	32.04%	31.17%
70	0.696	33.61%	33.01%	32.31%	31.11%	30.26%	30.17%	30.07%
75	0.746	30.73%	30.28%	29.75%	28.85%	28.54%	28.86%	28.71%
76	0.756	30.20%	29.79%	29.30%	28.46%	28.25%	28.58%	28.51%
77	0.766	29.67%	29.29%	28.84%	28.08%	27.97%	28.30%	28.31%
78	0.776	29.15%	28.80%	28.39%	27.69%	27.69%	28.02%	28.11%
79	0.786	28.62%	28.30%	27.93%	27.30%	27.41%	27.74%	27.91%
80	0.796	28.09%	27.81%	27.48%	26.91%	27.12%	27.46%	27.71%
81	0.806	27.61%	27.39%	27.13%	26.68%	26.88%	27.30%	27.59%
82	0.816	27.14%	26.97%	26.78%	26.44%	26.64%	27.14%	27.46%
83	0.826	26.66%	26.55%	26.42%	26.20%	26.39%	26.97%	27.34%
84	0.836	26.19%	26.14%	26.07%	25.97%	26.15%	26.81%	27.22%
85	0.846	25.71%	25.72%	25.72%	25.73%	25.91%	26.65%	27.09%
85.5	0.850	25.55%	25.67%	25.81%	26.05%	25.83%	26.55%	27.03%
86	0.855	25.38%	25.62%	25.90%	26.38%	25.75%	26.46%	26.96%
86.5	0.860	25.22%	25.57%	25.99%	26.70%	25.67%	26.37%	26.89%
87	0.865	25.05%	25.52%	26.08%	27.03%	25.59%	26.27%	26.83%
87.5	0.870	24.89%	25.48%	26.17%	27.35%	25.51%	26.18%	26.76%
88	0.875	24.75%	25.24%	25.81%	26.78%	25.40%	26.10%	26.68%
88.5	0.880	24.62%	25.00%	25.45%	26.21%	25.30%	26.03%	26.61%
89	0.885	24.49%	24.77%	25.09%	25.64%	25.19%	25.95%	26.53%
89.5	0.890	24.36%	24.53%	24.73%	25.07%	25.09%	25.88%	26.46%
90	0.895	24.23%	24.29%	24.37%	24.50%	24.98%	25.80%	26.38%
90.5	0.900	24.09%	24.16%	24.25%	24.40%	24.89%	25.72%	26.34%
91	0.905	23.95%	24.04%	24.14%	24.31%	24.79%	25.64%	26.30%
91.5	0.910	23.81%	23.91%	24.02%	24.21%	24.70%	25.55%	26.25%
92	0.915	23.67%	23.78%	23.90%	24.12%	24.60%	25.47%	26.21%
92.5	0.920	23.53%	23.65%	23.79%	24.02%	24.51%	25.39%	26.17%
93	0.925	23.40%	23.53%	23.67%	23.93%	24.45%	25.31%	26.09%
93.5	0.930	23.27%	23.40%	23.56%	23.83%	24.39%	25.23%	26.02%
94	0.935	23.13%	23.28%	23.45%	23.74%	24.32%	25.16%	25.94%
94.5	0.940	23.00%	23.15%	23.34%	23.65%	24.26%	25.08%	25.86%
95	0.945	22.87%	23.03%	23.22%	23.55%	24.20%	25.00%	25.79%
95.5	0.950	22.75%	22.92%	23.13%	23.47%	24.13%	24.97%	25.74%
96	0.955	22.63%	22.81%	23.03%	23.39%	24.06%	24.93%	25.70%
96.5	0.960	22.51%	22.70%	22.93%	23.31%	24.00%	24.89%	25.65%
97	0.965	22.39%	22.59%	22.83%	23.23%	23.93%	24.86%	25.61%
97.5	0.970	22.27%	22.48%	22.73%	23.15%	23.86%	24.82%	25.56%
98	0.975	22.17%	22.39%	22.64%	23.08%	23.79%	24.79%	25.53%
98.5	0.980	22.07%	22.30%	22.56%	23.01%	23.71%	24.75%	25.50%
99	0.985	21.98%	22.20%	22.47%	22.93%	23.64%	24.72%	25.47%
99.5	0.990	21.88%	22.11%	22.39%	22.86%	23.56%	24.68%	25.44%
100	0.995	21.78%	22.02%	22.31%	22.79%	23.49%	24.65%	25.41%
100.53	1.000	21.68%	21.93%	22.22%	22.71%	23.43%	24.60%	25.38%

101	1.005	21.60%	21.85%	22.14%	22.64%	23.38%	24.56%	25.34%
102	1.015	21.41%	21.67%	21.98%	22.50%	23.27%	24.47%	25.27%
103	1.025	21.23%	21.50%	21.81%	22.35%	23.16%	24.38%	25.20%
104	1.035	21.04%	21.32%	21.65%	22.20%	23.05%	24.29%	25.13%
105	1.044	20.86%	21.15%	21.48%	22.06%	22.94%	24.20%	25.06%
106	1.054	20.74%	21.03%	21.37%	21.96%	22.85%	24.14%	24.98%
107	1.064	20.62%	20.92%	21.27%	21.86%	22.76%	24.08%	24.91%
108	1.074	20.50%	20.81%	21.16%	21.76%	22.67%	24.02%	24.83%
109	1.084	20.39%	20.69%	21.05%	21.67%	22.58%	23.95%	24.76%
110	1.094	20.27%	20.58%	20.94%	21.57%	22.49%	23.89%	24.69%
111	1.104	20.24%	20.54%	20.90%	21.50%	22.42%	23.83%	24.64%
112	1.114	20.22%	20.51%	20.85%	21.44%	22.34%	23.77%	24.59%
113	1.124	20.19%	20.48%	20.81%	21.37%	22.26%	23.71%	24.54%
114	1.134	20.17%	20.44%	20.76%	21.31%	22.19%	23.65%	24.49%
115	1.144	20.15%	20.41%	20.72%	21.24%	22.11%	23.59%	24.44%
116	1.154	20.18%	20.43%	20.71%	21.20%	22.05%	23.55%	24.39%
117	1.164	20.22%	20.45%	20.71%	21.16%	21.99%	23.52%	24.33%
118	1.174	20.26%	20.47%	20.71%	21.13%	21.93%	23.48%	24.28%
119	1.184	20.29%	20.49%	20.71%	21.09%	21.88%	23.45%	24.23%
120	1.194	20.33%	20.50%	20.71%	21.05%	21.82%	23.41%	24.17%
125	1.243	20.46%	20.58%	20.72%	20.96%	21.70%	23.34%	24.21%
130	1.293	21.41%	21.33%	21.24%	21.08%	21.54%	23.27%	23.91%
135	1.343	22.80%	22.48%	22.11%	21.48%	21.56%	23.43%	23.80%
140	1.393	24.35%	23.75%	23.05%	21.85%	21.94%	23.74%	23.80%
145	1.442	25.97%	25.08%	24.04%	22.26%	21.97%	23.84%	23.91%
150	1.492	27.21%	26.23%	25.08%	23.13%	22.42%	23.83%	24.12%
155	1.542	28.66%	27.48%	26.10%	23.74%	22.98%	23.82%	24.42%
160	1.592	29.90%	28.63%	27.15%	24.62%	23.65%	23.82%	24.43%
165	1.641	31.87%	30.34%	28.54%	25.49%	23.98%	23.81%	24.78%
170	1.691	34.33%	32.68%	30.73%	27.42%	24.67%	23.80%	24.34%
175	1.741	35.17%	33.45%	31.44%	28.01%	25.35%	23.79%	24.95%
180	1.791	37.08%	35.16%	32.92%	29.09%	25.96%	23.79%	25.48%
185	1.840	37.03%	35.30%	33.26%	29.80%	27.23%	23.78%	25.48%
190	1.890	36.98%	36.35%	35.62%	34.37%	28.31%	23.77%	25.47%
195	1.940	36.93%	35.56%	33.96%	31.23%	28.34%	23.76%	25.47%

Source: Own calculation.